

2017-2018 学年 师大-中 学校 九 年级上半期 数学 试题详解

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11月

名师微点评

A 卷

一. 选择题.

1-5. C B C B A 6-10 D B A A D

二. 填空题.

11. -2; 12. -1; 13. $y = \frac{4}{x}$

14. 6 (射影定理).

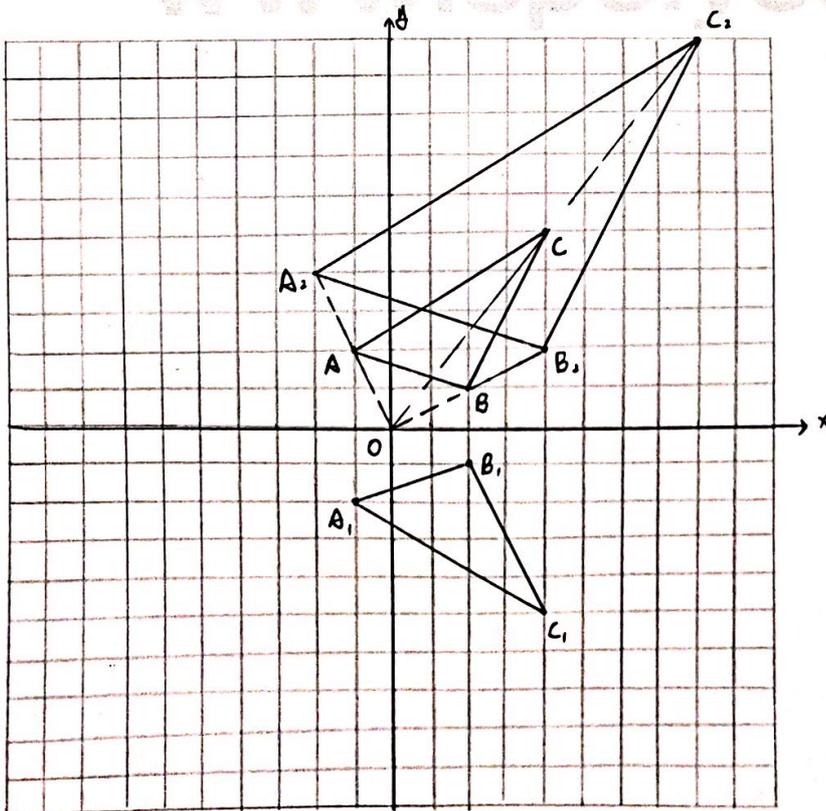
三. 解答题.

15. (1) $2\sqrt{3} + 9 - \sqrt{2}$;

(2) $x_1 = 2, x_2 = -8$;

16. 化简得: $\frac{a-1}{a-2}$, 代值得: $\frac{3+\sqrt{3}}{3}$;

17. (1) 如图
(2) 如图.
 $C_2(8, 10)$



位似中心: 对应点连线的交点.



解题老师:

18: (1) 30人;

(2) 列表:

	A	B	C	D
A		AB	AC	AD
B	BA		BC	BD
C	CA	CB		CD
D	DA	DB	DC	

∴ 共有12种等可能的结果,
恰好选中A, B的有2种

$$\therefore P(\text{选中A, B}) = \frac{2}{12} = \frac{1}{6}$$

答: 选中A, B两所学校概率为 $\frac{1}{6}$.

19: (1) 把A(1, n)代入 $y = -\frac{3}{x}$ 得 $n = -3 \therefore A(1, -3)$

把A(1, -3)代入 $y = x + m$ 得 $-3 = 1 + m$ 解得 $m = -4$

∴ 一次函数解析式为: $y = x - 4$

$$\text{联立 } \begin{cases} y = x - 4 \\ y = -\frac{3}{x} \end{cases} \text{ 得 } \begin{cases} x = 1 \\ y = -3 \end{cases} \text{ 或 } \begin{cases} x = 3 \\ y = -1 \end{cases}$$

$$\therefore B(3, -1) \quad \therefore BC = 1, OC = 3 \quad \therefore S_{\triangle OBC} = \frac{1}{2} BC \cdot OC = \frac{3}{2}$$

(2) $x < 0$ 或 $1 < x < 3$

20: (1) ∵ 点P, M, N分别为CD, DE, BC的中点

$$\therefore PM \parallel \frac{1}{2} CE, PN \parallel \frac{1}{2} BD$$

$$\therefore AB = AC, AD = AE$$

$$\therefore BD = CE$$

$$\therefore PM = PN$$

$$\text{又 } \angle A = 90^\circ \text{ 即 } BD \perp CE$$

$$\therefore PM \perp PN$$

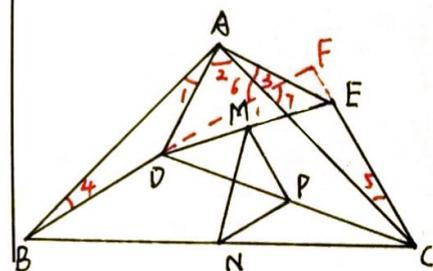
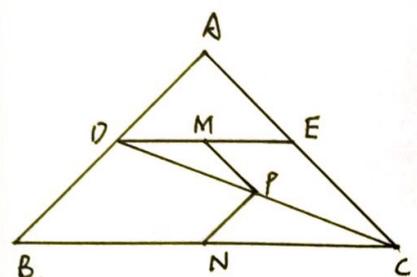
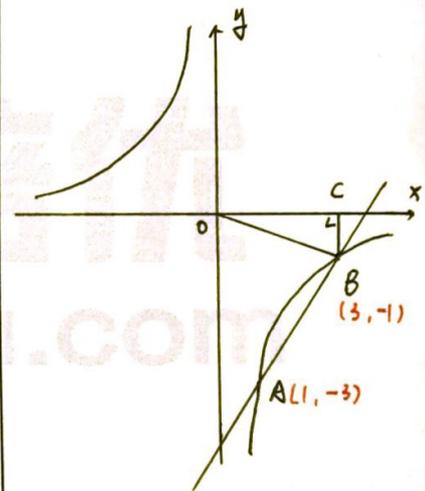
(2) $\triangle PMN$ 为等腰直角三角形. 理由如下:

∵ 点P, M, N分别为CD, DE, BC的中点

$$\therefore PM \parallel \frac{1}{2} CE, PN \parallel \frac{1}{2} BD$$

$$\text{又 } \angle 1 + \angle 2 = \angle BAC = 90^\circ, \angle 3 + \angle 2 = \angle DAE = 90^\circ$$

$$\therefore \angle 1 = \angle 3$$



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解题老师:

在 $\triangle ABD$ 和 $\triangle ACE$ 中

$$\begin{cases} AB=AC \\ \angle 1=\angle 3 \\ AD=AE \end{cases}$$

$\therefore \triangle ABD \cong \triangle ACE$ (SAS)

$\therefore BD=CE, \angle 4=\angle 5$

延长 BD, CE 交于 $F, \therefore \angle 6=\angle 7 \therefore \angle F=\angle BAC=90^\circ \therefore BD \perp CE$

$\therefore PM=PN$ 且 $PM \perp PN$

$\therefore \triangle PMN$ 为等腰直角三角形

(3) 要使 $\triangle PMN$ 面积最大, 即使 PN 或 PM 最大, 因为 $S_{\triangle PMN} = \frac{1}{2}PN^2 = \frac{1}{2}PM^2$

若使 PN 最大, 即使 BD 最大. $\therefore BD=2PN$ 又 $\triangle ADE$ 可绕 A 点在平面内任意旋转. 则 $AB-AD \leq BD \leq AB+AD$ 即 $6 \leq BD \leq 14$

$\therefore BD_{\max}=14, \therefore PN_{\max}=7 \therefore S_{\triangle PMN_{\max}} = \frac{1}{2} \times 7^2 = \frac{49}{2}$

B 卷:

一. 填空题.

21: $\frac{20}{3} \text{cm}^2$ 面积之比等于相似比的平方

22: $\frac{21}{4}$ 由韦达定理 $\begin{cases} x_1+x_2=5 \\ x_1x_2=a \end{cases}$ 又 $x_1^2-x_2^2=(x_1-x_2)(x_1+x_2)=10$

则 $x_1-x_2=2$ 又 $x_1x_2 = \frac{(x_1+x_2)^2 - (x_1-x_2)^2}{4} = \frac{21}{4}$

23: -12 由题意得: $4m=-6n \Rightarrow n=-\frac{2}{3}m$. 设 t 为公共根.

则 $t^2+4t+m=0, t^2-6t+n=0 \Rightarrow t = \frac{n-m}{10} = \frac{-\frac{2}{3}m-m}{10} = -\frac{m}{6}$

\therefore 把 $t = -\frac{m}{6}$ 代入 $t^2+4t+m=0$ 得 $(-\frac{m}{6})^2 + 4(-\frac{m}{6}) + m = 0$

解得 $m_1=0, m_2=-12$ 又 $m \neq 0 \therefore m = -12$

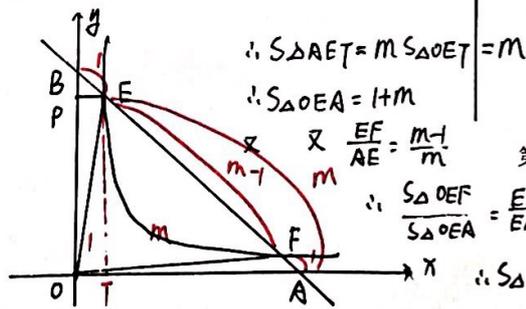
24: $2, \frac{m^2-1}{m}$

$S_{\triangle OEP} = \frac{|k|}{2} = 1, k > 0 \Rightarrow k=2$

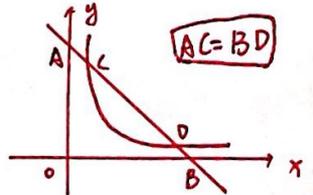
作 $ET \perp x$ 轴于 T . 则 $S_{\triangle OET} = \frac{|k|}{2} = 1$

又 $BE=AF \Rightarrow BF=AE$

$\therefore \frac{BE}{BF} = \frac{BE}{AE} = \frac{1}{m} \therefore \frac{OT}{AT} = \frac{BE}{AE} = \frac{1}{m}$



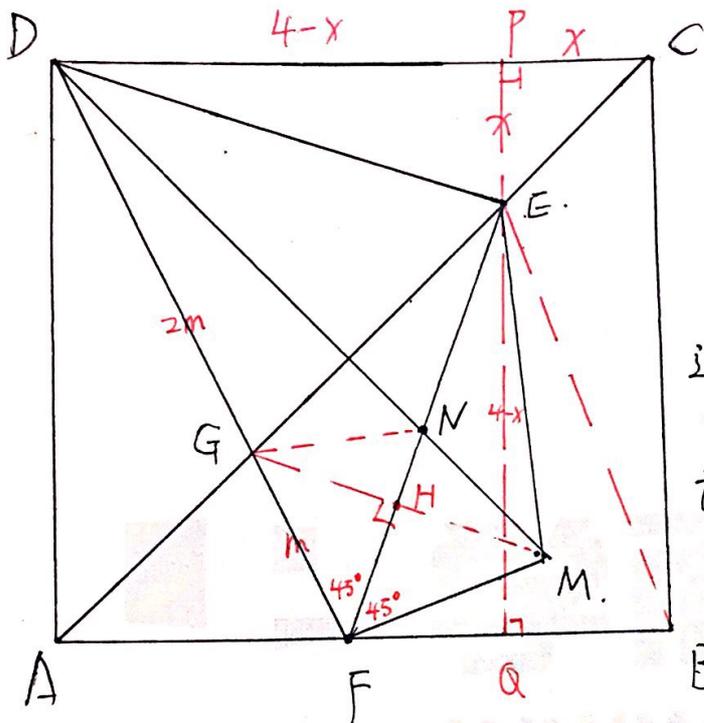
B 24 考查反比例函数与一次函数相交的结论.



第 页, 共 页



B25.



$$\frac{5\sqrt{2}}{2} + \frac{\sqrt{10}}{2}$$

解析:

过E作AD平行线交DC于P, 交AB于Q, 连EB, NG, MG
设PE=x, 则PC=x.

$$DP = EQ = 4 - x$$

$$B \quad \triangle EQF \cong \triangle DPE (AAS)$$

$$\Downarrow$$

$$DE = EF$$

$$\triangle DEC \cong \triangle BEC (SAS) \Rightarrow DE = EB = EF. \therefore EQ \perp FB. \therefore FQ = QB = 1$$

$$\therefore PE = PC = 1. \quad PD = 3 \quad DE = EF = \sqrt{10}. \quad \triangle DEF \text{ 为等腰} \text{Rt} \triangle$$

$$DC \parallel AB \quad \therefore \frac{FG}{GD} = \frac{AF}{DC} = \frac{1}{2} \quad (F \text{ 为中点}) \quad \therefore FG = \frac{1}{3} DF$$

$$DF = \sqrt{AF^2 + AD^2} = 2\sqrt{5}, \quad \therefore FG = \frac{2\sqrt{5}}{3} \quad \text{由对称, } \triangle GHF \text{ 与 } \triangle MHF \text{ 均为等腰} \text{Rt} \triangle.$$

$$FH = \frac{\sqrt{2}}{2} FG = \frac{\sqrt{10}}{3}, \quad \angle EDF = \angle NDA = 45^\circ \quad \therefore \angle EDN = \angle FDA$$

$$\therefore \frac{EN}{ED} = \frac{1}{2} \quad EN = \frac{DE}{2} = \frac{\sqrt{10}}{2}$$

$$NH = EF - FH - EN = \frac{\sqrt{10}}{6} \quad \therefore NG = \sqrt{NH^2 + GH^2} = \frac{5\sqrt{2}}{6}$$

$$EG = AC - AG - EC. \quad EC = \sqrt{2}. \quad AG = \frac{1}{3} AC = \frac{4\sqrt{2}}{3} \quad \therefore EG = \frac{5\sqrt{2}}{3}$$

$$\therefore C_{\triangle ENM} = C_{\triangle ENG} = EN + NG + EG = \frac{\sqrt{10}}{2} + \frac{5\sqrt{2}}{6} + \frac{5\sqrt{2}}{3} = \frac{5\sqrt{2} + \sqrt{10}}{2}$$

直角比较多的复杂图形中, 需找出其中尽量多

相等的锐角.

它们所在的Rt△中, 三边比相等.



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B 26.

(1) 解: 设 2015 年这种礼盒进价为 x 元/盒.则 2017 年进价为 $(x-11)$ 元/盒.

有: $\frac{3500}{x} = \frac{2700}{x-11}$ 解得 $x=35$.

经检验, $x=35$ 为原方程的解.

答: 2015 年进价为 35 元/盒.

(2). 设年增长率为 a . 销售量为 $\frac{3500}{35} = 100$.

依题意.

$$100 \cdot (65-35) \cdot (1+a)^2 = (60-35+11) \cdot 100.$$

$$a = 0.2 \text{ 或 } a = -2.2 \text{ (舍)}$$

答: 年增长率为 20%



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B27

解: (1). 代入点 $A(8,1)$, 得 $m=8 \times 1=8 \quad \therefore y = \frac{8}{x}$

则 $n \cdot 8=8, n=1, \therefore$ 设 $AB: y=kx+b$

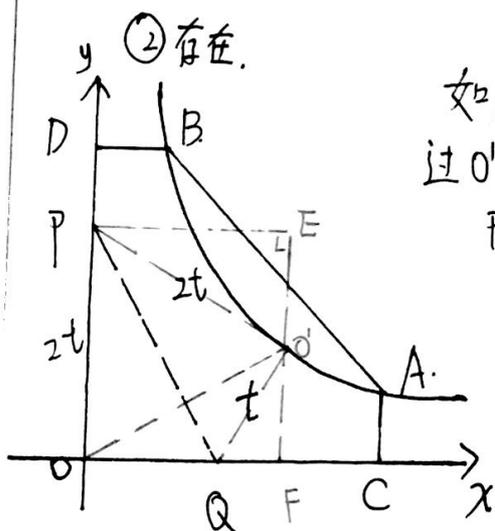
代入 $A(8,1), B(1,8)$ 得 $\begin{cases} k=-1 \\ b=9 \end{cases}$

$\therefore AB: y=-x+9$

(2) ① 由已知: $OP=2t, OQ=t$

则当 P 在 OD 上运动时, $S = \frac{1}{2} OP \cdot OQ = t^2 \quad (0 < t \leq 4)$

则当 P 在 DB 上运动时, $S = \frac{1}{2} OQ \cdot OD = 4t \quad (4 < t \leq 4.5)$



② 存在. 如图, O' 在反比例图象上时.

过 O' 作 $EO'F \parallel y$ 轴, 交 y 轴于 F , 交 x 轴平行线 PE 于 E 点.

$$PO' = PO = 2t \quad QO' = QO = t$$

$$\triangle PEQ \sim \triangle O'FQ$$

设 QF 为 b , $O'F$ 为 a .

$$PE = t + b, \quad O'E = 2t - a$$

$$\frac{t+b}{a} = \frac{2t-a}{b} = 2, \quad a = \frac{4}{5}t, \quad b = \frac{3}{5}t$$

$$\text{则 } O' \left(\frac{8}{5}t, \frac{4}{5}t \right)$$

$$\text{有: } \frac{8}{5}t \cdot \frac{4}{5}t = 8 \quad t = \pm \frac{5}{2} \quad \text{舍去负值, } t = \frac{5}{2}$$

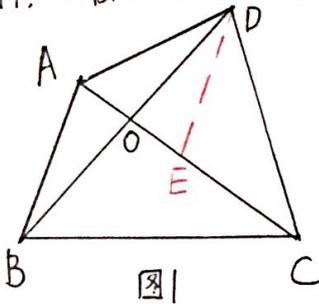
$\therefore O'(4, 2)$ 当 $t = \frac{5}{2}$ 时, O' 恰好落于反比例

图象上.



B 28

(1) 角平分线: $\angle BAD + \angle ACB = 180^\circ$, 理由如下:



在 $\triangle BAD$ 中: $\angle BAD + \angle ABD + \angle ADB = 180^\circ$
 $\therefore \angle ABD + \angle ADB = \angle ACB$
 $\therefore \angle BAD + \angle ACB = 180^\circ$

(2) 如图1, 作 $DE \parallel AB$ 交 AC 于 E . 则 $\angle DEA = \angle BAE$, $\angle OBA = \angle ODE$.

$\because OB = OD, \therefore \triangle OAB \cong \triangle OED$ (AAS) $\therefore DE \parallel AB, OA = OE$.

$\because OC = OA + AB$ 且 $OA = OE, \therefore EC = AB = DE$.

$\because \angle EDA + \angle DAB = 180^\circ, \angle BAD + \angle ACB = 180^\circ \therefore \angle EDA = \angle ACB$

而 $\angle AED = \angle CAB \therefore \triangle EAD \sim \triangle ABC$.

$$\therefore \frac{ED}{AC} = \frac{AE}{AB} = \frac{AD}{BC} = \frac{m}{n}$$

$$\therefore \frac{ED}{AC} = \frac{a}{a+2b} = \frac{AE}{AB} = \frac{2b}{a}$$

$$\therefore \frac{m}{n} = \frac{\sqrt{5}-1}{2}$$

设 $AB = DE = CE = a, OA = OE = b$.

(设 a 为 1)
 $a^2 = 2b \cdot (a+2b)$ 解得 $\frac{2b}{a} = \frac{\sqrt{5}-1}{2}$

作 $DE \parallel AB$ 交 AC 于 E .

由 (2) 知

$DE = EC = AB = a, \therefore \angle DCA = \angle DCA'$

$\therefore \angle EDC = \angle A'CD \therefore DE \parallel AB \parallel A'C$

$\triangle EAD \sim \triangle ABC$.

$\therefore \angle DAE = \angle ABC = \angle DA'C$.

$\therefore \angle DA'C + \angle A'CB = 180^\circ \therefore DA' \parallel BC$

$\therefore \triangle PA'D \sim \triangle PBC$.

$$\therefore \frac{A'D}{BC} = \frac{PD}{PC} = \frac{\sqrt{5}-1}{2} \therefore \frac{PC}{CD} = \frac{2}{\sqrt{5}+1} \therefore PC = CD \cdot \frac{2}{\sqrt{5}+1} = |$$

第(2), (3)问做

法本质是截长,

形成 $\square ADEB$.

从而利用第(1)问

构造等角, 构造相似

(3)

