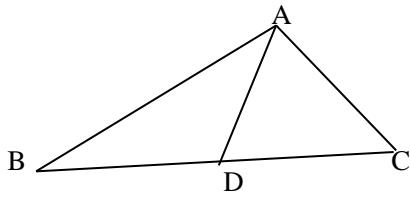


1. 已知: $AB=4$, $AC=2$, D 是 BC 中点, AD 是整数, 求 AD



解: 延长 AD 到 E , 使 $AD=DE$

$\because D$ 是 BC 中点

$\therefore BD=DC$

在 $\triangle ACD$ 和 $\triangle BDE$ 中

$AD=DE$

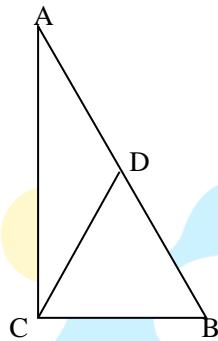
$$\angle BDE=\angle ADC \quad BD=DC$$

$$\therefore \triangle ACD \cong \triangle BDE \quad \therefore AC=BE=2$$

\because 在 $\triangle ABE$ 中 $AB-BE < AE < AB+BE$

$$\because AB=4 \text{ 即 } 4-2 < 2AD < 4+2 \quad 1 < AD < 3 \quad \therefore AD=2$$

2. 已知: D 是 AB 中点, $\angle ACB=90^\circ$, 求证: $CD=\frac{1}{2}AB$



延长 CD 与 P , 使 D 为 CP 中点。连接 AP, BP

$\because DP=DC, DA=DB$

$\therefore ACBP$ 为平行四边形

又 $\angle ACB=90^\circ$

\therefore 平行四边形 $ACBP$ 为矩形

$\therefore AB=CP=1/2AB$

3. 已知: $BC=DE$, $\angle B=\angle E$, $\angle C=\angle D$, F 是 CD 中点, 求证: $\angle 1=\angle 2$

证明: 连接 BF 和 EF

$$\because BC=ED, CF=DF, \angle BCF=\angle EDF$$

\therefore 三角形 BCF 全等于三角形 EDF (边角边)

$$\therefore BF=EF, \angle CBF=\angle DEF$$

连接 BE

在三角形 BEF 中, $BF=EF$

$$\therefore \angle EBF=\angle BEF.$$

$$\because \angle ABC=\angle AED.$$

$$\therefore \angle ABE=\angle AEB.$$

$$\therefore AB=AE.$$

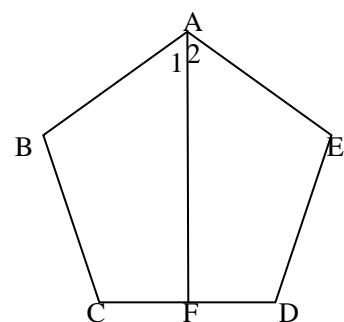
在三角形 ABF 和三角形 AEF 中

$$AB=AE, BF=EF,$$

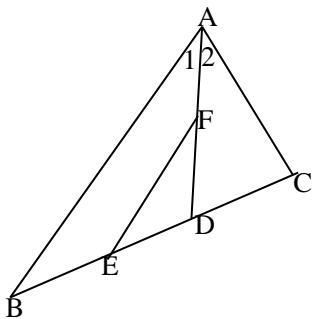
$$\angle ABF=\angle ABE+\angle EBF=\angle AEB+\angle BEF=\angle AEF$$

\therefore 三角形 ABF 和三角形 AEF 全等。

$$\therefore \angle BAF=\angle EAF (\angle 1=\angle 2).$$



4. 已知: $\angle 1=\angle 2$, $CD=DE$, $EF//AB$, 求证: $EF=AC$



过 C 作 $CG \parallel EF$ 交 AD 的延长线于点 G

$CG \parallel EF$, 可得, $\angle EFD = \angle CGD$

$DE = DC$

$\angle FDE = \angle GDC$ (对顶角)

$\therefore \triangle EFD \cong \triangle CGD$

$EF = CG$

$\angle CGD = \angle EFD$

又, $EF \parallel AB$

$\therefore \angle EFD = \angle 1$

$\angle 1 = \angle 2$

$\therefore \angle CGD = \angle 2$

$\therefore \triangle AGC$ 为等腰三角形,

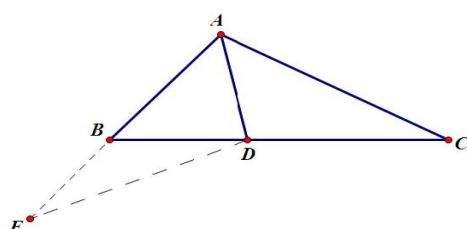
$AC = CG$

又 $EF = CG$

$\therefore EF = AC$

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5. 已知: AD 平分 $\angle BAC$, $AC = AB + BD$, 求证: $\angle B = 2\angle C$



证明: 延长 AB 取点 E, 使 $AE = AC$, 连接 DE

$\because AD$ 平分 $\angle BAC$

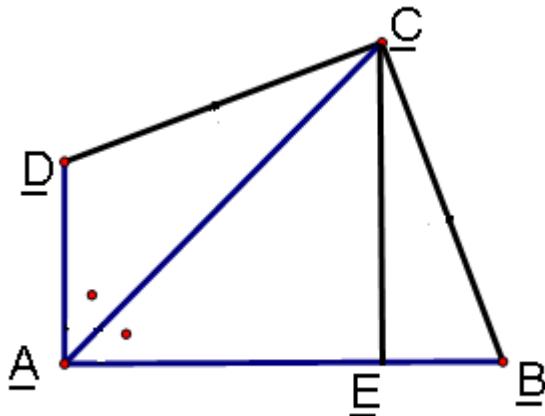
$\therefore \angle EAD = \angle CAD$

$\because AE = AC$, $AD = AD$

$\therefore \triangle AED \cong \triangle ACD$ (SAS)

$\therefore \angle E = \angle C$
 $\because AC = AB + BD$
 $\therefore AE = AB + BD$
 $\therefore AE = AB + BE$
 $\therefore BD = BE$
 $\therefore \angle BDE = \angle E$
 $\because \angle ABC = \angle E + \angle BDE$
 $\therefore \angle ABC = 2\angle E$
 $\therefore \angle ABC = 2\angle C$

6. 已知: AC 平分 $\angle BAD$, $CE \perp AB$, $\angle B + \angle D = 180^\circ$, 求证: $AE = AD + BE$

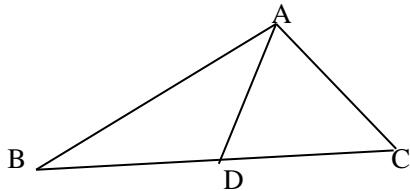


训练营

证明:

在 AE 上取 F , 使 $EF = EB$, 连接 CF
 $\because CE \perp AB$
 $\therefore \angle CEB = \angle CEF = 90^\circ$
 $\because EB = EF, CE = CE,$
 $\therefore \triangle CEB \cong \triangle CEF$
 $\therefore \angle B = \angle CFE$
 $\because \angle B + \angle D = 180^\circ, \angle CFE + \angle CFA = 180^\circ$
 $\therefore \angle D = \angle CFA$
 $\because AC$ 平分 $\angle BAD$
 $\therefore \angle DAC = \angle FAC$
 $\because AC = AC$
 $\therefore \triangle ADC \cong \triangle AFC$ (SAS)
 $\therefore AD = AF$
 $\therefore AE = AF + FE = AD + BE$

7. 已知: $AB = 4$, $AC = 2$, D 是 BC 中点, AD 是整数, 求 AD



解：延长AD到E,使 $AD=DE$

$\because D$ 是BC中点

$\therefore BD=DC$

在 $\triangle ACD$ 和 $\triangle BDE$ 中

$AD=DE$

$\angle BDE=\angle ADC$

$BD=DC$

$\therefore \triangle ACD \cong \triangle BDE$

$\therefore AC=BE=2$

\because 在 $\triangle ABE$ 中

$AB-BE < AE < AB+BE$

$\therefore AB=4$

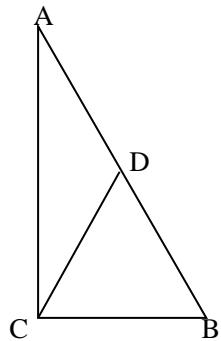
即 $4-2 < 2AD < 4+2$

$1 < AD < 3$

$\therefore AD=2$



8. 已知： D 是 AB 中点， $\angle ACB=90^\circ$ ，求证： $CD=\frac{1}{2}AB$



解：延长AD到E,使 $AD=DE$

$\because D$ 是BC中点

$\therefore BD=DC$

在 $\triangle ACD$ 和 $\triangle BDE$ 中

$AD=DE$

$$\angle BDE = \angle ADC$$

$$BD = DC$$

$$\therefore \triangle ACD \cong \triangle BDE$$

$$\therefore AC = BE = 2$$

∴ 在 $\triangle ABE$ 中

$$AB - BE < AE < AB + BE$$

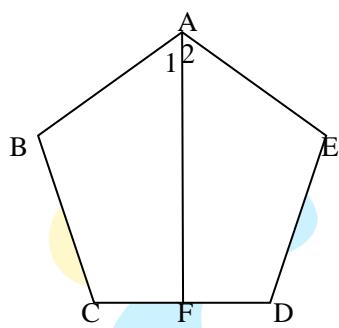
$$\therefore AB = 4$$

$$\text{即 } 4 - 2 < 2AD < 4 + 2$$

$$1 < AD < 3$$

$$\therefore AD = 2$$

9. 已知: $BC = DE$, $\angle B = \angle E$, $\angle C = \angle D$, F 是 CD 中点, 求证: $\angle 1 = \angle 2$



证明: 连接 BF 和 EF。

$$\because BC = ED, CF = DF, \angle BCF = \angle EDF.$$

∴ 三角形 BCF 全等于三角形 EDF(边角边)。

$$\therefore BF = EF, \angle CBF = \angle DEF.$$

连接 BE。

在三角形 BEF 中, $BF = EF$ 。

$$\therefore \angle EBF = \angle BEF.$$

又 ∵ $\angle ABC = \angle AED$ 。

$$\therefore \angle ABE = \angle AEB.$$

$$\therefore AB = AE.$$

在三角形 ABF 和三角形 AEF 中,

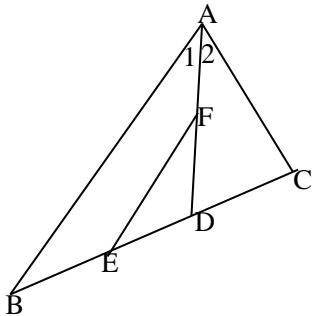
$$AB = AE, BF = EF,$$

$$\angle ABF = \angle ABE + \angle EBF = \angle AEB + \angle BEF = \angle AEF.$$

∴ 三角形 ABF 和三角形 AEF 全等。

$$\therefore \angle BAF = \angle EAF (\angle 1 = \angle 2).$$

10. 已知: $\angle 1 = \angle 2$, $CD = DE$, $EF \parallel AB$, 求证: $EF = AC$



过 C 作 $CG \parallel EF$ 交 AD 的延长线于点 G

$CG \parallel EF$, 可得, $\angle EFD = \angle CGD$

$DE = DC$

$\angle FDE = \angle GDC$ (对顶角)

$\therefore \triangle EFD \cong \triangle CGD$

$EF = CG$

$\angle CGD = \angle EFD$

又 $EF \parallel AB$

$\therefore \angle EFD = \angle 1$

$\angle 1 = \angle 2$

$\therefore \angle CGD = \angle 2$

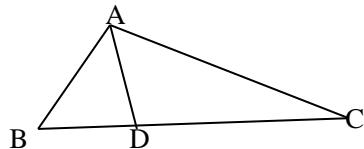
$\therefore \triangle AGC$ 为等腰三角形,

$AC = CG$

又 $EF = CG$

$\therefore EF = AC$

11. 已知: AD 平分 $\angle BAC$, $AC = AB + BD$, 求证: $\angle B = 2\angle C$



证明: 延长 AB 取点 E, 使 $AE = AC$, 连接 DE

$\because AD$ 平分 $\angle BAC$

$\therefore \angle EAD = \angle CAD$

$\because AE = AC$, $AD = AD$

$\therefore \triangle AED \cong \triangle ACD$ (SAS)

$\therefore \angle E = \angle C$

$\because AC = AB + BD$

$\therefore AE = AB + BD$

$\therefore AE = AB + BE$

$\therefore BD = BE$

$\therefore \angle BDE = \angle E$

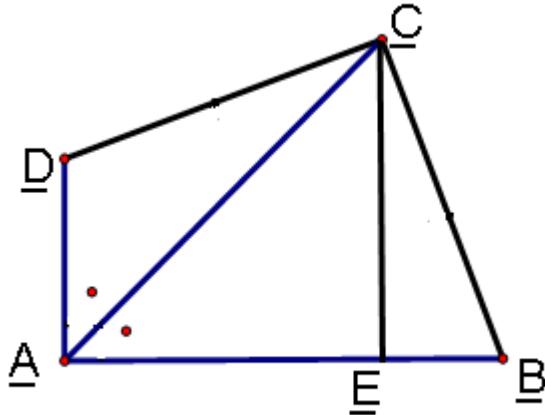
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$$\because \angle ABC = \angle E + \angle BDE$$

$$\therefore \angle ABC = 2\angle E$$

$$\therefore \angle ABC = 2\angle C$$

12. 已知: AC 平分 $\angle BAD$, $CE \perp AB$, $\angle B + \angle D = 180^\circ$, 求证: $AE = AD + BE$



在 AE 上取 F, 使 $EF = EB$, 连接 CF

$$\because CE \perp AB$$

$$\therefore \angle CEB = \angle CEF = 90^\circ$$

$$\because EB = EF, CE = CE,$$

$$\therefore \triangle CEB \cong \triangle CEF$$

$$\therefore \angle B = \angle CFE$$

$$\because \angle B + \angle D = 180^\circ, \angle CFE + \angle CFA = 180^\circ$$

$$\therefore \angle D = \angle CFA$$

$$\because AC \text{ 平分 } \angle BAD$$

$$\therefore \angle DAC = \angle FAC$$

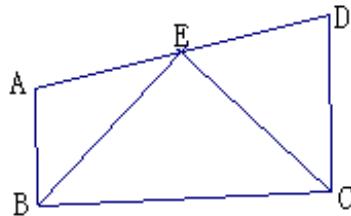
$$\text{又} \because AC = AC$$

$$\therefore \triangle ADC \cong \triangle AFC (\text{SAS})$$

$$\therefore AD = AF$$

$$\therefore AE = AF + FE = AD + BE$$

12. 如图, 四边形 ABCD 中, $AB \parallel DC$, BE 、 CE 分别平分 $\angle ABC$ 、 $\angle BCD$, 且点 E 在 AD 上。求证: $BC = AB + DC$ 。

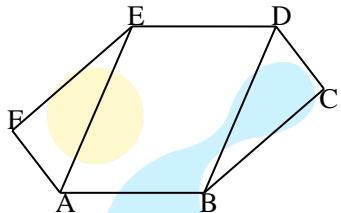


在 BC 上截取 $BF = AB$, 连接 EF

$\because BE$ 平分 $\angle ABC$

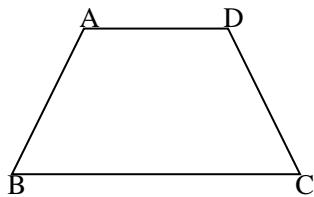
$\therefore \angle ABE = \angle FBE$
 又 $\because BE = BE$
 $\therefore \triangle ABE \cong \triangle FBE$ (SAS)
 $\therefore \angle A = \angle BFE$
 $\because AB \parallel CD$
 $\therefore \angle A + \angle D = 180^\circ$
 $\therefore \angle BFE + \angle CFE = 180^\circ$
 $\therefore \angle D = \angle CFE$
 又 $\because \angle DCE = \angle FCE$
 CE 平分 $\angle BCD$
 $CE = CE$
 $\therefore \triangle DCE \cong \triangle FCE$ (AAS)
 $\therefore CD = CF$
 $\therefore BC = BF + CF = AB + CD$

13. 已知: $AB \parallel ED$, $\angle EAB = \angle BDE$, $AF = CD$, $EF = BC$, 求证: $\angle F = \angle C$



$AB \parallel ED$, 得: $\angle EAB + \angle AED = \angle BDE + \angle ABD = 180^\circ$ 度,
 $\therefore \angle EAB = \angle BDE$,
 $\therefore \angle AED = \angle ABD$,
 \therefore 四边形 $ABDE$ 是平行四边形。
 \therefore 得: $AE = BD$,
 $\because AF = CD, EF = BC$,
 \therefore 三角形 AEF 全等于三角形 DBC ,
 $\therefore \angle F = \angle C$ 。

14. 已知: $AB = CD$, $\angle A = \angle D$, 求证: $\angle B = \angle C$



证明: 设线段 AB, CD 所在的直线交于 E , (当 $AD < BC$ 时, E 点是射线 BA, CD 的交点, 当 $AD > BC$ 时, E 点是射线 AB, DC 的交点)。则:

$\triangle AED$ 是等腰三角形。

$\therefore AE = DE$

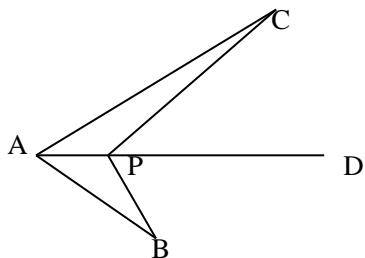
而 $AB=CD$

$\therefore BE=CE$ (等量加等量, 或等量减等量)

$\therefore \triangle BEC$ 是等腰三角形

$\therefore \angle B=\angle C.$

15. P 是 $\angle BAC$ 平分线 AD 上一点, $AC>AB$, 求证: $PC-PB<AC-AB$



在 AC 上取点 E,

使 $AE=AB$ 。

$\because AE=AB$

$AP=AP$

$\angle EAP=\angle BAE$,

$\therefore \triangle EAP \cong \triangle BAP$

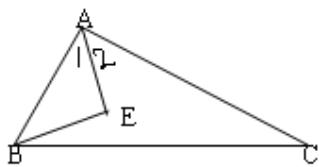
$\therefore PE=PB$ 。

$PC<EC+PE$

$\therefore PC<(AC-AE)+PB$

$\therefore PC-PB<AC-AB$ 。

16. 已知 $\angle ABC=3\angle C$, $\angle 1=\angle 2$, $BE \perp AE$, 求证: $AC-AB=2BE$



证明:

在 AC 上取一点 D, 使得角 $DBC=\angle C$

$\because \angle ABC=3\angle C$

$\therefore \angle ABD=\angle ABC-\angle DBC=3\angle C-\angle C=2\angle C$;

$\therefore \angle ADB=\angle C+\angle DBC=2\angle C$;

$\therefore AB=AD$

$\therefore AC-AB=AC-AD=CD=BD$

在等腰三角形 ABD 中, AE 是角 BAD 的角平分线,

$\therefore AE \perp BD$

$\because BE \perp AE$

\therefore 点 E 一定在直线 BD 上,

在等腰三角形 ABD 中, $AB=AD$, $AE \perp BD$

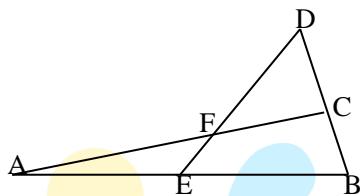
\therefore 点 E 也是 BD 的中点

$\therefore BD=2BE$

$\because BD=CD=AC-AB$

$\therefore AC-AB=2BE$

17. 已知, E 是 AB 中点, $AF=BD$, $BD=5$, $AC=7$, 求 DC



\because 作 $AG \parallel BD$ 交 DE 延长线于 G

$\therefore AGE$ 全等 BDE

$\therefore AG=BD=5$

$\therefore AGF \sim CDF$

$AF=AG=5$

$\therefore DC=CF=2$

18. 如图, 在 $\triangle ABC$ 中, $BD=DC$, $\angle 1=\angle 2$, 求证: $AD \perp BC$.



C 于点 E ,

$\therefore \triangle BDC$ 是等腰三角形

DCB

$\therefore \angle DBC+\angle 1=\angle DCB+\angle 2$

即 $\angle ABC=\angle ACB$

$\therefore \triangle ABC$ 是等腰三角形

$\therefore AB=AC$

在 $\triangle ABD$ 和 $\triangle ACD$ 中

{ $AB=AC$

$\angle 1=\angle 2$

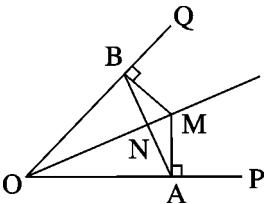
$BD=DC$

$\therefore \triangle ABD$ 和 $\triangle ACD$ 是全等三角形 (边角边)

$\therefore \angle BAD = \angle CAD$
 $\therefore AE$ 是 $\triangle ABC$ 的中垂线
 $\therefore AE \perp BC$
 $\therefore AD \perp BC$

19. 如图, OM 平分 $\angle POQ$, $MA \perp OP, MB \perp OQ$, A, B 为垂足, AB 交 OM 于点 N .

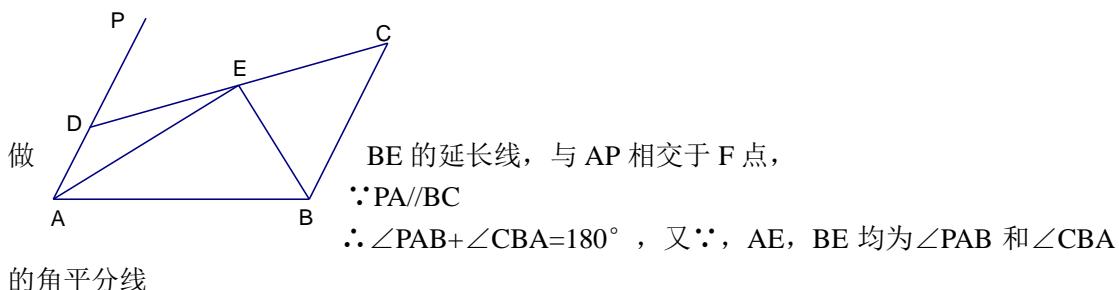
求证: $\angle OAB = \angle OBA$



证明:

$\because OM$ 平分 $\angle POQ$
 $\therefore \angle POM = \angle QOM$
 $\because MA \perp OP, MB \perp OQ$
 $\therefore \angle MAO = \angle MBO = 90^\circ$
 $\because OM = OM$
 $\therefore \triangle AOM \cong \triangle BOM$ (AAS)
 $\therefore OA = OB$
 $\because ON = ON$
 $\therefore \triangle AON \cong \triangle BON$ (SAS)
 $\therefore \angle OAB = \angle OBA, \angle ONA = \angle ONB$
 $\because \angle ONA + \angle ONB = 180^\circ$
 $\therefore \angle ONA = \angle ONB = 90^\circ$
 $\therefore OM \perp AB$

20. (5 分) 如图, 已知 $AD \parallel BC$, $\angle PAB$ 的平分线与 $\angle CBA$ 的平分线相交于 E , CE 的连线交 AP 于 D . 求证: $AD+BC=AB$.



的角平分线

$\therefore \angle EAB + \angle EBA = 90^\circ \therefore \angle AEB = 90^\circ$, EAB 为直角三角形

在三角形 ABF 中, $AE \perp BF$, 且 AE 为 $\angle FAB$ 的角平分线

\therefore 三角形 FAB 为等腰三角形, $AB = AF, BE = EF$

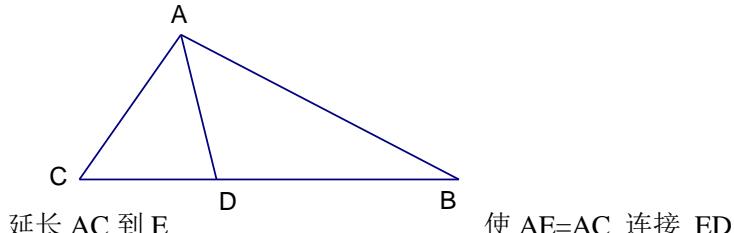
在三角形 DEF 与三角形 BEC 中,

$\angle EBC = \angle DFE$, 且 $BE = EF, \angle DEF = \angle CEB$,

\therefore 三角形 DEF 与三角形 BEC 为全等三角形, $\therefore DF = BC$

$\therefore AB = AF = AD + DF = AD + BC$

21. 如图, $\triangle ABC$ 中, AD 是 $\angle CAB$ 的平分线, 且 $AB=AC+CD$, 求证: $\angle C=2\angle B$



$$\because AB=AC+CD$$

$$\therefore CD=CE$$

可得 $\angle B=\angle E$

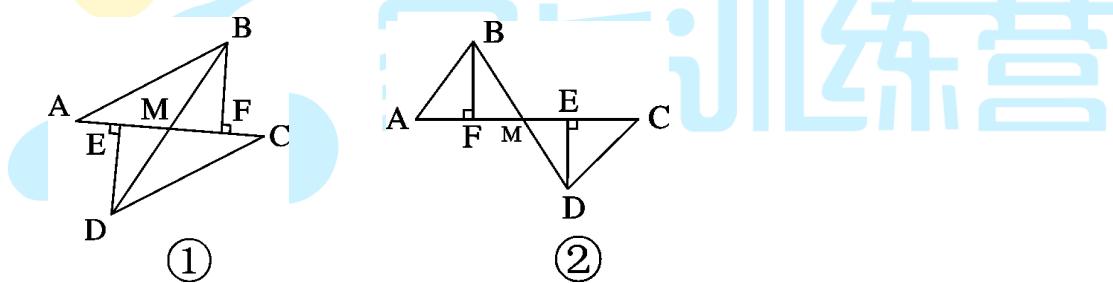
$\triangle CDE$ 为等腰

$$\angle ACB=2\angle B$$

22. (6 分) 如图①, E 、 F 分别为线段 AC 上的两个动点, 且 $DE \perp AC$ 于 E , $BF \perp AC$ 于 F , 若 $AB=CD$, $AF=CE$, BD 交 AC 于点 M .

(1) 求证: $MB=MD$, $ME=MF$

(2) 当 E 、 F 两点移动到如图②的位置时, 其余条件不变, 上述结论能否成立? 若成立请给予证明; 若不成立请说明理由.



(1) 连接 BE , DF .

$\because DE \perp AC$ 于 E , $BF \perp AC$ 于 F ,

$\therefore \angle DEC=\angle BFA=90^\circ$, $DE \parallel BF$,

在 $\text{Rt}\triangle DEC$ 和 $\text{Rt}\triangle BFA$ 中,

$\because AF=CE$, $AB=CD$,

$\therefore \text{Rt}\triangle DEC \cong \text{Rt}\triangle BFA$ (HL),

$\therefore DE=BF$.

\therefore 四边形 $BEDF$ 是平行四边形.

$\therefore MB=MD$, $ME=MF$;

(2) 连接 BE , DF .

$\because DE \perp AC$ 于 E , $BF \perp AC$ 于 F ,

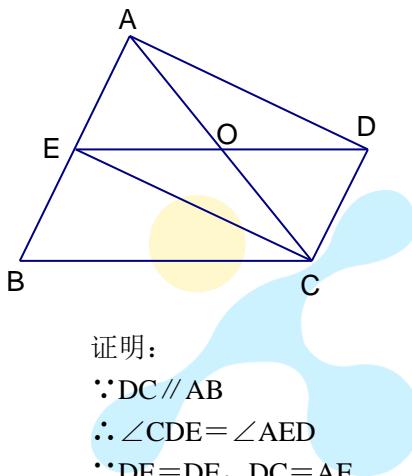
$\therefore \angle DEC=\angle BFA=90^\circ$, $DE \parallel BF$,

在 $\text{Rt}\triangle DEC$ 和 $\text{Rt}\triangle BFA$ 中,

$\because AF=CE$, $AB=CD$,
 $\therefore Rt\triangle DEC \cong Rt\triangle BFA$ (HL),
 $\therefore DE=BF$.
 \therefore 四边形 BEDF 是平行四边形.
 $\therefore MB=MD$, $ME=MF$.

23. 已知: 如图, $DC \parallel AB$, 且 $DC=AE$, E 为 AB 的中点,

- (1) 求证: $\triangle AED \cong \triangle EBC$.
- (2) 观看图前, 在不添辅助线的情况下, 除 $\triangle EBC$ 外, 请再写出两个与 $\triangle AED$ 的面积相等的三角形. (直接写出结果, 不要求证明):

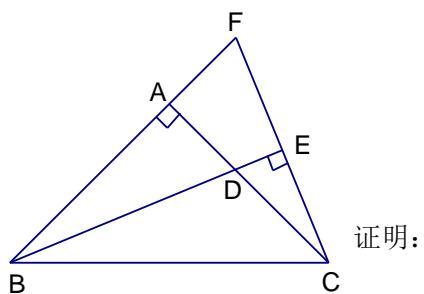


证明:

$\because DC \parallel AB$
 $\therefore \angle CDE = \angle AED$
 $\because DE = DE$, $DC = AE$
 $\therefore \triangle AED \cong \triangle EDC$
 $\because E$ 为 AB 中点
 $\therefore AE = BE$
 $\therefore BE = DC$
 $\because DC \parallel AB$
 $\therefore \angle DCE = \angle BEC$
 $\because CE = CE$
 $\therefore \triangle EBC \cong \triangle EDC$
 $\therefore \triangle AED \cong \triangle EBC$

24. (7 分) 如图, $\triangle ABC$ 中, $\angle BAC = 90^\circ$, $AB = AC$, BD 是 $\angle ABC$ 的平分线, BD 的延长线垂直于过 C 点的直线于 E , 直线 CE 交 BA 的延长线于 F .

求证: $BD = 2CE$.



证明:

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$\because \angle CEB = \angle CAB = 90^\circ$

\therefore ABCE 四点共圆

$\therefore \angle ABE = \angle CBE$

$\therefore AE = CE$

$\therefore \angle ECA = \angle EAC$

取线段 BD 的中点 G，连接 AG，则：AG=BG=DG

$\therefore \angle GAB = \angle ABG$

而： $\angle ECA = \angle GBA$ （同弧上的圆周角相等）

$\therefore \angle ECA = \angle EAC = \angle GBA = \angle GAB$

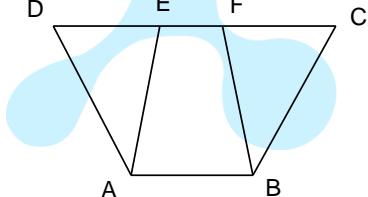
而： $AC = AB$

$\therefore \triangle AEC \cong \triangle AGB$

$\therefore EC = BG = DG$

$\therefore BE = 2CE$

25、如图： $DF = CE$, $AD = BC$, $\angle D = \angle C$ 。求证： $\triangle AED \cong \triangle BFC$ 。



证明： $\because DF = CE$,

$\therefore DF - EF = CE - EF$,

即 $DE = CF$,

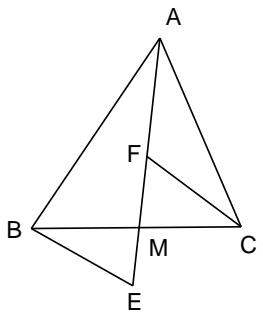
在 $\triangle AED$ 和 $\triangle BFC$ 中,

$\because AD = BC$, $\angle D = \angle C$, $DE = CF$

$\therefore \triangle AED \cong \triangle BFC$ (SAS)

26、(10 分) 如图：AE、BC 交于点 M, F 点在 AM 上, $BE \parallel CF$, $BE = CF$ 。

求证：AM 是 $\triangle ABC$ 的中线。



证明：

$$\because BE \parallel CF$$

$$\therefore \angle E = \angle CFM, \angle EBM = \angle FCM$$

$$\because BE = CF$$

$$\therefore \triangle BEM \cong \triangle CFM$$

$$\therefore BM = CM$$

$\therefore AM$ 是 $\triangle ABC$ 的中线。

27、(10分) 如图：在 $\triangle ABC$ 中， $BA=BC$ ， D 是 AC 的中点。求证： $BD \perp AC$ 。



$\because \triangle ABD$ 和 $\triangle BCD$ 的三条边都相等

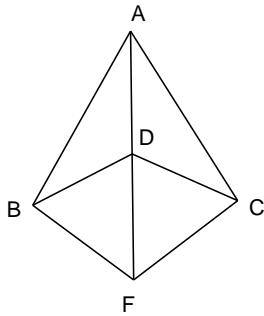
$$\therefore \triangle ABD \cong \triangle BCD$$

$$\therefore \angle ADB = \angle CDB$$

$$\therefore \angle ADB = \angle CDB = 90^\circ$$

$$\therefore BD \perp AC$$

28、(10分) $AB=AC$ ， $DB=DC$ ， F 是 AD 的延长线上的一点。求证： $BF=CF$



在 $\triangle ABD$ 与 $\triangle ACD$ 中

$$AB=AC$$

$$BD=DC$$

$$AD=AD$$

$$\therefore \triangle ABD \cong \triangle ACD$$

$$\therefore \angle ADB = \angle ADC$$

$$\therefore \angle BDF = \angle FDC$$

在 $\triangle BDF$ 与 $\triangle FDC$ 中

$$BD=DC$$

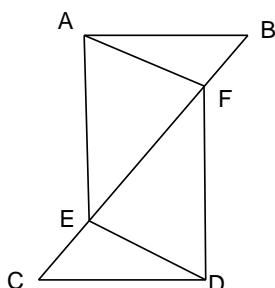
$$\angle BDF = \angle FDC$$

$$DF=DF$$

$$\therefore \triangle FBD \cong \triangle FCD$$

$$\therefore BF=FC$$

29、(12 分) 如图: $AB=CD$, $AE=DF$, $CE=FB$ 。求证: $AF=DE$ 。



$$\because AB=CD$$

$$AE=DF,$$

$$CE=FB$$

$$CE+EF=EF+FB$$

$$\therefore \triangle ABE \cong \triangle CDF$$

$$\therefore \angle DCB = \angle ABF$$

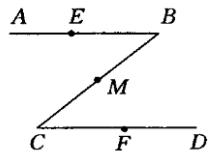
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$$AB=DC \quad BF=CE$$

$$\triangle ABF \cong \triangle CDE$$

$$\therefore AF=DE$$

30. 公园里有一条“Z”字形道路 $ABCD$, 如图所示, 其中 $AB \parallel CD$, 在 AB , CD , BC 三段路旁各有一只小石凳 E , F , M , 且 $BE=CF$, M 在 BC 的中点, 试说明三只石凳 E , F , M 恰好在一条直线上.



证明: 连接 EF

$$\because AB \parallel CD$$

$$\therefore \angle B = \angle C$$

$$\because M \text{ 是 } BC \text{ 中点}$$

$$\therefore BM = CM$$

在 $\triangle BEM$ 和 $\triangle CFM$ 中

$$BE = CF$$

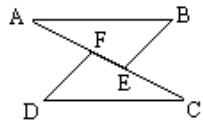
$$\angle B = \angle C$$

$$BM = CM$$

$$\therefore \triangle BEM \cong \triangle CFM \text{ (SAS)}$$

$$\therefore CF = BE$$

31. 已知: 点 A , F , E , C 在同一条直线上, $AF=CE$, $BE \parallel DF$, $BE=DF$. 求证: $\triangle ABE \cong \triangle CDF$.



(第2题)

$$\because AF = CE, FE = EF.$$

$$\therefore AE = CF.$$

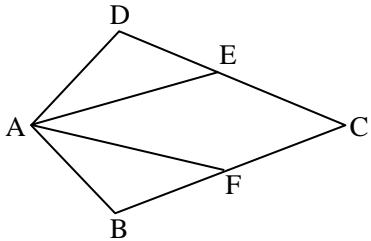
$$\because DF \parallel BE,$$

$$\therefore \angle AEB = \angle CFD \text{ (两直线平行, 内错角相等)}$$

$$\therefore BE = DF$$

$$\therefore \triangle ABE \cong \triangle CDF \text{ (SAS)}$$

32. 已知: 如图所示, $AB = AD$, $BC = DC$, E 、 F 分别是 DC 、 BC 的中点, 求证: $AE = AF$.



连接 BD ;

$$\because AB=AD \quad BC=DC$$

$$\therefore \angle ADB=\angle ABD \quad \angle CDB=\angle ABD; \text{两角相加, } \angle ADC=\angle ABC;$$

$$\because BC=DC \quad E\backslash F \text{ 是中点}$$

$$\therefore DE=BF;$$

$$\because AB=AD \quad DE=BF$$

$$\angle ADC=\angle ABC$$

$$\therefore AE=AF.$$

33. 如图, 在四边形 $ABCD$ 中, E 是 AC 上的一点, $\angle 1=\angle 2$, $\angle 3=\angle 4$, 求证: $\angle 5=\angle 6$.



证明:

在 $\triangle ADC$, $\triangle ABC$ 中

$$\because AC=AC, \quad \angle BAC=\angle DAC, \quad \angle BCA=\angle DCA$$

$$\therefore \triangle ADC \cong \triangle ABC \text{ (两角加一边)}$$

$$\therefore AB=AD, \quad BC=CD$$

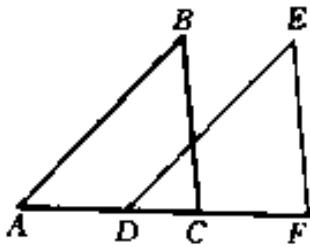
在 $\triangle DEC$ 与 $\triangle BEC$ 中

$$\angle BCA=\angle DCA, \quad CE=CE, \quad BC=CD$$

$$\therefore \triangle DEC \cong \triangle BEC \text{ (两边夹一角)}$$

$$\therefore \angle DEC=\angle BEC$$

34. 已知 $AB//DE$, $BC//EF$, D , C 在 AF 上, 且 $AD=CF$, 求证: $\triangle ABC \cong \triangle DEF$.



$$\because AD = DF$$

$$\therefore AC = DF$$

$$\because AB \parallel DE$$

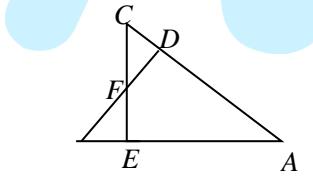
$$\therefore \angle A = \angle EDF$$

$$\text{又} \because BC \parallel EF$$

$$\therefore \angle F = \angle BCA$$

$$\therefore \triangle ABC \cong \triangle DEF \text{ (ASA)}$$

35. 已知: 如图, $AB=AC$, $BD \perp AC$, $CE \perp AB$, 垂足分别为 D 、 E , BD 、 CE 相交于点 F , 求证: $BE=CD$.



证明:

$$\because BD \perp AC$$

$$\therefore \angle BDC = 90^\circ$$

$$\because CE \perp AB$$

$$\therefore \angle BEC = 90^\circ$$

$$\therefore \angle BDC = \angle BEC = 90^\circ$$

$$\because AB = AC$$

$$\therefore \angle DCB = \angle EBC$$

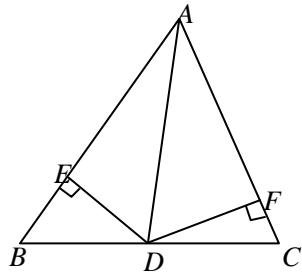
$$\therefore BC = BC$$

$$\therefore Rt\triangle BDC \cong Rt\triangle BEC \text{ (AAS)}$$

$$\therefore BE = CD$$

36、如图，在 $\triangle ABC$ 中， AD 为 $\angle BAC$ 的平分线， $DE \perp AB$ 于 E ， $DF \perp AC$ 于 F 。

求证： $DE=DF$.



证明：

$\because AD$ 是 $\angle BAC$ 的平分线

$\therefore \angle EAD = \angle FAD$

$\because DE \perp AB$, $DF \perp AC$

$\therefore \angle BFD = \angle CFD = 90^\circ$

$\therefore \angle AED$ 与 $\angle AFD = 90^\circ$

在 $\triangle AED$ 与 $\triangle AFD$ 中

$\angle EAD = \angle FAD$

$AD = AD$

$\angle AED = \angle AFD$

$\therefore \triangle AED \cong \triangle AFD$ (AAS)

$\therefore AE = AF$

在 $\triangle AEO$ 与 $\triangle AFO$ 中

$\angle EAO = \angle FAO$

$AO = AO$

$AE = AF$

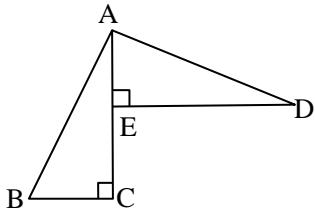
$\therefore \triangle AEO \cong \triangle AFO$ (SAS)

$\therefore \angle AOE = \angle AOF = 90^\circ$

$\therefore AD \perp EF$

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37. 已知：如图， $AC \perp BC$ 于 C , $DE \perp AC$ 于 E , $AD \perp AB$ 于 A , $BC = AE$. 若 $AB = 5$, 求 AD 的长?



$$\because AD \perp AB$$

$$\therefore \angle BAC = \angle ADE$$

又 $\because AC \perp BC$ 于 C , $DE \perp AC$ 于 E

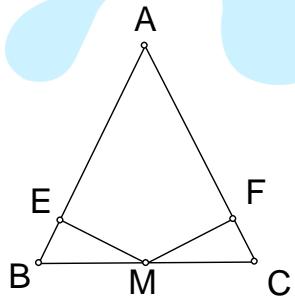
根据三角形角度之和等于180度

$$\therefore \angle ABC = \angle DAE$$

$$\because BC = AE, \triangle ABC \cong \triangle DAE \text{ (ASA)}$$

$$\therefore AD = AB = 5$$

38. 如图： $AB=AC$, $ME \perp AB$, $MF \perp AC$, 垂足分别为 E 、 F , $ME=MF$. 求证： $MB=MC$



证明：

$$\because AB = AC$$

$$\therefore \angle B = \angle C$$

$$\because ME \perp AB, MF \perp AC$$

$$\therefore \angle BEM = \angle CFM = 90^\circ$$

在 $\triangle BME$ 和 $\triangle CMF$ 中

$$\because \angle B = \angle C, \angle BEM = \angle CFM = 90^\circ, ME = MF$$

$$\therefore \triangle BME \cong \triangle CMF \text{ (AAS)}$$

$$\therefore MB = MC.$$

39. 如图，给出五个等量关系：① $AD = BC$ ② $AC = BD$ ③ $CE = DE$ ④ $\angle D = \angle C$
⑤ $\angle DAB = \angle CBA$. 请你以其中两个为条件，另三个中的一个为结论，推出一个正确的结论（只需写出一种情况），并加以证明。

已知：① $AD = BC$, ⑤ $\angle DAB = \angle CBA$

求证: $\triangle DAB \cong \triangle CBA$

证明: $\because AD=BC, \angle DAB=\angle CBA$

又 $\because AB=AB$

$\therefore \triangle DAB \cong \triangle CBA$

40. 在 $\triangle ABC$ 中, $\angle ACB = 90^\circ$, $AC = BC$, 直线 MN 经过点 C , 且 $AD \perp MN$ 于 D ,

$BE \perp MN$ 于 E . (1) 当直线 MN 绕点 C 旋转到图 1 的位置时, 求证: ① $\triangle ADC \cong \triangle CEB$;

② $DE = AD + BE$;

(2) 当直线 MN 绕点 C 旋转到图 2 的位置时, (1) 中的结论还成立吗? 若成立, 请给出证明; 若不成立, 说明理由.

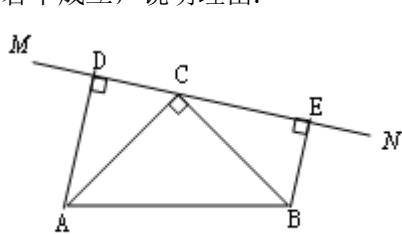


图 1

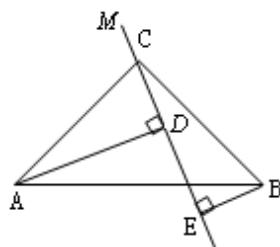


图 2

(1)

① $\because \angle ADC = \angle ACB = \angle BEC = 90^\circ$,

$\therefore \angle CAD + \angle ACD = 90^\circ$, $\angle BCE + \angle CBE = 90^\circ$, $\angle ACD + \angle BCE = 90^\circ$.

$\therefore \angle CAD = \angle BCE$.

$\because AC = BC$,

$\therefore \triangle ADC \cong \triangle CEB$.

② $\because \triangle ADC \cong \triangle CEB$,

$\therefore CE = AD$, $CD = BE$.

$\therefore DE = CE + CD = AD + BE$.

(2) $\because \angle ADC = \angle CEB = \angle ACB = 90^\circ$,

$\therefore \angle ACD = \angle CBE$.

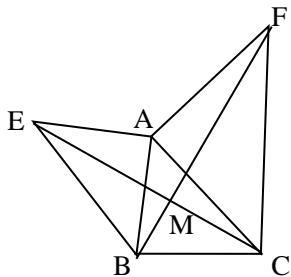
又 $\because AC = BC$,

$\therefore \triangle ACD \cong \triangle CBE$.

$\therefore CE = AD$, $CD = BE$.

$\therefore DE = CE - CD = AD - BE$

41. 如图所示, 已知 $AE \perp AB$, $AF \perp AC$, $AE = AB$, $AF = AC$. 求证: (1) $EC = BF$; (2) $EC \perp BF$



(1) $\because AE \perp AB, AF \perp AC,$

$\therefore \angle BAE = \angle CAF = 90^\circ,$

$\therefore \angle BAE + \angle BAC = \angle CAF + \angle BAC,$

即 $\angle EAC = \angle BAF,$

在 $\triangle ABF$ 和 $\triangle AEC$ 中，

$\because AE = AB, \angle EAC = \angle BAF, AF = AC,$

$\therefore \triangle ABF \cong \triangle AEC (\text{SAS}),$

$\therefore EC = BF;$

(2) 如图, 根据 (1), $\triangle ABF \cong \triangle AEC,$

$\therefore \angle AEC = \angle ABF,$

$\because AE \perp AB,$

$\therefore \angle BAE = 90^\circ,$

$\therefore \angle AEC + \angle ADE = 90^\circ,$

$\because \angle ADE = \angle BDM$ (对顶角相等),

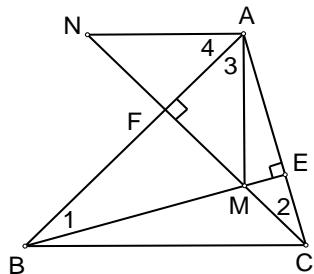
$\therefore \angle ABF + \angle BDM = 90^\circ,$

在 $\triangle BDM$ 中, $\angle BMD = 180^\circ - \angle ABF - \angle BDM = 180^\circ - 90^\circ = 90^\circ,$

$\therefore EC \perp BF.$

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42. 如图: $BE \perp AC, CF \perp AB, BM = AC, CN = AB$ 。求证: (1) $AM = AN$; (2) $AM \perp AN$ 。



证明:

(1)

$\because BE \perp AC, CF \perp AB$

$$\therefore \angle ABM + \angle BAC = 90^\circ, \quad \angle ACN + \angle BAC = 90^\circ$$

$$\therefore \angle ABM = \angle ACN$$

$$\because BM = AC, \quad CN = AB$$

$$\therefore \triangle ABM \cong \triangle NAC$$

$$\therefore AM = AN$$

(2)

$$\because \triangle ABM \cong \triangle NAC$$

$$\therefore \angle BAM = \angle N$$

$$\because \angle N + \angle BAN = 90^\circ$$

$$\therefore \angle BAM + \angle BAN = 90^\circ$$

$$\text{即 } \angle MAN = 90^\circ$$

$$\therefore AM \perp AN$$

43. 如图,已知 $\angle A=\angle D$, $AB=DE$, $AF=CD$, $BC=EF$.求证: $BC \parallel EF$



在 $\triangle ABF$ 和 $\triangle CDE$ 中

$$AB = DE$$

$$\angle A = \angle D$$

$$AF = CD$$

$$\therefore \triangle ABF \equiv \triangle CDE \text{ (边角边)}$$

$$\therefore FB = CE$$

在四边形 BCEF 中

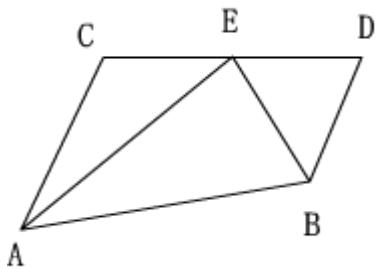
$$FB = CE$$

$$BC = EF$$

\therefore 四边形 BCEF 是平行四边形

$$\therefore BC \parallel EF$$

44. 如图,已知 $AC \parallel BD$, EA 、 EB 分别平分 $\angle CAB$ 和 $\angle DBA$, CD 过点 E , 则 AB 与 $AC+BD$ 相等吗? 请说明理由



在 AB 上取点 N ,使得 $AN=AC$

$\because \angle CAE=\angle EAN$

$\therefore AE$ 为公共,

$\therefore \triangle CAE \cong \triangle EAN$

$\therefore \angle ANE=\angle ACE$

又 $\because AC$ 平行 BD

$\therefore \angle ACE+\angle BDE=180$

而 $\angle ANE+\angle ENB=180$

$\therefore \angle ENB=\angle BDE$

$\angle NBE=\angle EBN$

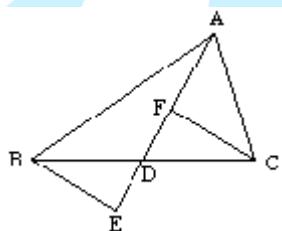
$\because BE$ 为公共边

$\therefore \triangle EBN \cong \triangle EBD$

$\therefore BD=BN$

$\therefore AB=AN+BN=AC+BD$

45、(10分) 如图,已知: AD 是 BC 上的中线 ,且 $DF=DE$. 求证: $BE \parallel CF$.



证明:

$\because AD$ 是 $\triangle ABC$ 的中线

$BD=CD$

$\because DF=DE$ (已知)

$\angle BDE=\angle FDC$

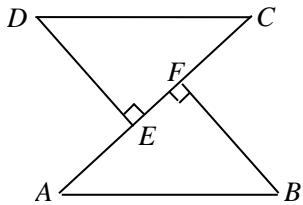
$\therefore \triangle BDE \cong \triangle FDC$

则 $\angle EBD=\angle FCD$

$\therefore BE \parallel CF$ (内错角相等, 两直线平行)。

46、(10分)已知: 如图, $AB=CD$, $DE \perp AC$, $BF \perp AC$, E , F 是垂足, $DE=BF$.

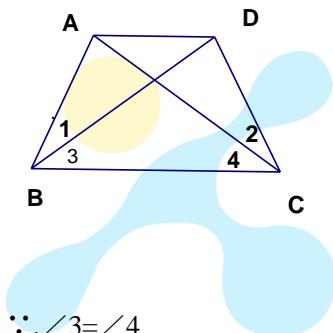
求证: $AB \parallel CD$.



证明:

$$\begin{aligned} & \because DE \perp AC, BF \perp AC \\ & \therefore \angle CED = \angle AFB = 90^\circ \\ & \text{又} \because AB = CD, BF = DE \\ & \therefore \text{Rt } \triangle ABF \cong \text{Rt } \triangle CDE \text{ (HL)} \\ & \therefore AF = CE \\ & \angle BAF = \angle DCE \\ & \therefore AB \parallel CD \end{aligned}$$

47、(10分)如图, 已知 $\angle 1=\angle 2$, $\angle 3=\angle 4$, 求证: $AB=CD$

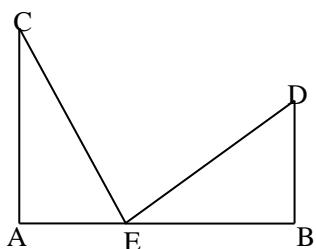


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$$\begin{aligned} & \because \angle 3 = \angle 4 \\ & \therefore OB = OC \\ & \text{在 } \triangle AOB \text{ 和 } \triangle DOC \text{ 中} \\ & \quad \angle 1 = \angle 2 \\ & \quad OB = OC \\ & \quad \angle AOB = \angle DOC \\ & \quad \triangle AOB \cong \triangle DOC \\ & \therefore AO = DO \quad AO + OC = DO + OB \quad AC = DB \end{aligned}$$

$$\begin{aligned} & \text{在 } \triangle ACB \text{ 和 } \triangle DBC \text{ 中} \\ & \quad AC = DB \\ & \quad \angle 3 = \angle 4 \\ & \quad BC = CB \\ & \quad \triangle ACB \cong \triangle DBC \\ & \therefore AB = CD \end{aligned}$$

48、(10分)如图, 已知 $AC \perp AB$, $DB \perp AB$, $AC = BE$, $AE = BD$, 试猜想线段 CE 与 DE 的大小与位置关系, 并证明你的结论.



$CE > DE$ 。当 $\angle AEB$ 越小，则 DE 越小。

证明：

过 D 作 AE 平行线与 AC 交于 F，连接 FB

由已知条件知 $AFDE$ 为平行四边形， $ABEC$ 为矩形，且 $\triangle DFB$ 为等腰三角形。

RT $\triangle BAE$ 中， $\angle AEB$ 为锐角，即 $\angle AEB < 90^\circ$

$\because DF \parallel AE \quad \therefore \angle FDB = \angle AEB < 90^\circ$

$\triangle DFB$ 中 $\angle DFB = \angle DBF = (180^\circ - \angle FDB)/2 > 45^\circ$

RT $\triangle AFB$ 中， $\angle FBA = 90^\circ - \angle DBF < 45^\circ$

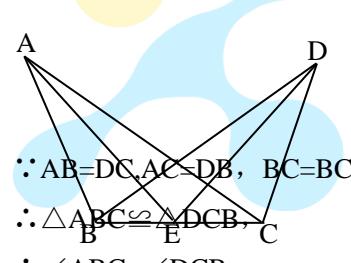
$\angle AFB = 90^\circ - \angle FBA > 45^\circ$

$\therefore AB > AF$

$\because AB = CE \quad AF = DE$

$\therefore CE > DE$

49、(10 分)如图，已知 $AB = DC$, $AC = DB$, $BE = CE$, 求证: $AE = DE$.



又 $\because BE = CE$, $AB = DC$

$\therefore \triangle ABE \cong \triangle DCE$

$\therefore AE = DE$

50. 如图 9 所示， $\triangle ABC$ 是等腰直角三角形， $\angle ACB = 90^\circ$ ， AD 是 BC 边上的中线，过 C 作 AD 的垂线，交 AB 于点 E，交 AD 于点 F，求证： $\angle ADC = \angle BDE$.

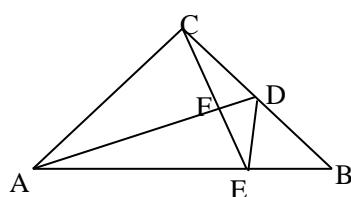


图 9

作 $CG \perp AB$,交 AD 于 H ,
则 $\angle ACH=45^\circ$, $\angle BCH=45^\circ$
 $\because \angle CAH=90^\circ - \angle CDA$, $\angle BCE=90^\circ - \angle CDA \therefore \angle CAH=\angle BCE$
又 $\because AC=CB$, $\angle ACH=\angle B=45^\circ$
 $\therefore \triangle ACH \cong \triangle CBE$, $\therefore CH=BE$
又 $\because \angle DCH=\angle B=45^\circ$, $CD=DB$
 $\therefore \triangle CFD \cong \triangle BED$
 $\therefore \angle ADC=\angle BDE$



