

2018 年深圳市中考数学试卷答案

一、选择题

1. A 2. B 3. C 4. D 5. A
6. B 7. D 8. A 9. A 10. D

【解析】设三角板与圆的切点为 C

连接 OA , OB

由切线长定理可知: $AB = AC$

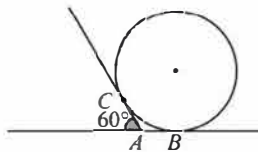
$\therefore OA$ 平分 $\angle BAC$

$\therefore \angle OAB = 60^\circ$

在 $\text{Rt}\triangle OAB$ 中

$$OA = \frac{AB}{\tan \angle OAB} = 3\sqrt{3}$$

\therefore 直径为 $6\sqrt{3}$



11. C

【解析】开口向下: $a < 0$

对称轴为 $x = 1$; $b > 0$ $-\frac{b}{2a} = 1$

与 y 轴交点在正半轴: $c > 0$

$\therefore A: abc < 0$ A 错误

B: $-\frac{b}{2a} = 1$ $2a + b = 0$ B 错误

C: 当 $x = 1$ 时, $y < 0$, 即 $a - b + c < 0$ 将 $b = -2a$ 代入, 得 $3a + c < 0$, C 正确

D: $ax^2 + bx + c - 3 = 0$, 即 $ax^2 + bx + c = 3$ 表示该二次函数与 $y = 3$ 的交点个数显然仅有一个交点, D 错误.

12. B

【解析】①显然 AO 与 BO 不一定相等

故 $\triangle AOP$ 与 $\triangle BOP$ 不一定全等

①错误

②延长 BP , 交 x 轴于点 E

延长 AP , 交 y 轴于点 F

$\therefore AP \parallel x$ 轴, $BP \parallel y$ 轴

\therefore 四边形 $OEFP$ 为矩形

$$S_{\triangle EOP} = S_{\triangle FOP}$$

$$\therefore S_{\triangle BOE} = S_{\triangle AOF} = k$$

$$\therefore S_{\triangle AOP} = S_{\triangle BOP}$$

②正确

③过 P 作 $PM \perp BO$, 垂足为 M

过 P 作 $PN \perp AO$, 垂足为 N

$$\therefore S_{\triangle AOP} = \frac{1}{2} AO \cdot PN$$

$$S_{\triangle BOP} = \frac{1}{2} BO \cdot PM$$

$$S_{\triangle AOP} = S_{\triangle BOP}$$

$$AO = BO$$

$$\therefore PM = PN$$

∴ PO 平分 ∠AOB

OP 为 ∠AOB 的平分线

③ 正确

④ 法一: 设 $P(a, b)$, 则 $B\left(a, \frac{12}{a}\right)$ $A\left(\frac{12}{b}, b\right)$

$$S_{\triangle BOP} = \frac{1}{2} BP \cdot EO$$

$$= \frac{1}{2} \times \left(\frac{12}{a} - b\right) \cdot a = 4$$

$$12 - ab = 8$$

$$ab = 4$$

$$S_{\triangle ABP} = \frac{1}{2} AP \cdot BP$$

$$= \frac{1}{2} \times \left(\frac{12}{a} - b\right) \times \left(\frac{12}{b} - a\right)$$

$$= \frac{1}{2} \times \left(\frac{144}{ab} - 12 - 12 + ab\right)$$

$$= 8$$

④ 错误

④ 法二: $S_{\triangle BOE} = \frac{k}{2} = 6$

$$\therefore S_{\triangle BOP} = 4$$

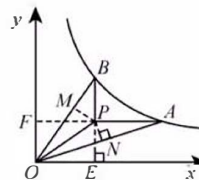
$$\therefore S_{\triangle POE} = 2$$

$$\therefore \frac{BP}{PE} = \frac{2}{1}$$

$$\therefore \frac{S_{\triangle ABP}}{S_{\triangle AEP}} = \frac{BP}{PE} = \frac{2}{1}$$

$$S_{\triangle AEP} = S_{\triangle AOP}$$

$$\therefore S_{\triangle ABP} = 8$$



二、填空题

13. $(a+3)(a-3)$

14. $\frac{1}{2}$

15. 8

【解析】易知: $\triangle ACE \cong \triangle FAB$

$$\therefore CE = AB = 4$$

$$S_{\triangle ABC} = \frac{1}{2} \times 4 \times 4 = 8$$

16.

【解析】法一: $\because AE, BD$ 分别平分 $\angle CAB$ 和 $\angle CBA$

$$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4$$

$$\because \angle C = 90^\circ$$

$$\therefore \angle 2 + \angle 3 = 45^\circ$$

$$\therefore \angle AFD = 45^\circ$$

过 D 作 $DG \perp AE$, 垂足为 G 在 $\text{Rt}\triangle DFG$ 中

$$\angle DFG = 45^\circ, DF = \sqrt{2}$$

$$\therefore DG = FG = 1$$

在 $\text{Rt}\triangle ADG$ 中

$$AG = AF - GF = 3$$

$$AD = \sqrt{AG^2 + DG^2} = \sqrt{10}$$

过 F 作 $FH \perp AC$, 垂足为 H

$$\text{设 } DH = a, \text{ 则 } FH^2 = DF^2 - DH^2 = 2 - a^2$$

在 $\text{Rt}\triangle AHF$ 中

$$AH^2 + HF^2 = AF^2$$

$$\text{即 } (\sqrt{10} + a)^2 + 2 - a^2 = 16$$

$$a = \frac{\sqrt{10}}{5}$$

$$CH = HF = \frac{2\sqrt{10}}{5}$$

$$\therefore AC = AD + DH + CH = \frac{8\sqrt{10}}{5}$$

法二: $\because AE, BD$ 分别平分 $\angle CAB$ 和 $\angle CBA$

$$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4$$

$$\because \angle C = 90^\circ$$

$$\therefore \angle 2 + \angle 3 = 45^\circ$$

$$\therefore \angle AFD = 45^\circ$$

过 D 作 $DG \perp AE$, 垂足为 G 在 $\text{Rt}\triangle DFG$ 中

$$\angle DFG = 45^\circ, DF = \sqrt{2}$$

$$\therefore DG = FG = 1$$

在 $\text{Rt}\triangle ADG$ 中

$$AG = AF - GF = 3$$

$$AD = \sqrt{AG^2 + DG^2} = \sqrt{10}$$

连接 CF , 易知 CF 平分 $\angle ACB$

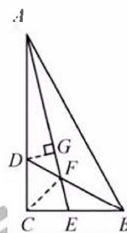
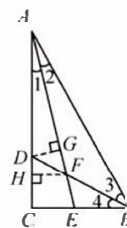
$$\therefore \angle ACF = 45^\circ$$

$$\because \angle ACF = \angle AFD = 45^\circ$$

$$\therefore \triangle ACF \sim \triangle AFD$$

$$\therefore \frac{AC}{AF} = \frac{AF}{AD}$$

$$AC = \frac{AF^2}{AD} = \frac{16}{\sqrt{10}} = \frac{8\sqrt{10}}{5}$$



法三: $\because AE, BD$ 分别平分 $\angle CAB$ 和 $\angle CBA$

$$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4$$

$$\because \angle C = 90^\circ$$

$$\therefore \angle 2 + \angle 3 = 45^\circ$$

$$\therefore \angle AFD = 45^\circ$$

过 D 作 $DG \perp AE$, 垂足为 G

在 $Rt\triangle DFG$ 中

$$\angle DFG = 45^\circ, DF = \sqrt{2}$$

$$\therefore DG = FG = 1$$

在 $Rt\triangle ADG$ 中

$$AG = AF - GF = 3$$

$$\tan \angle DAG = \frac{1}{3}$$

过 F 作 $FM \perp AC$, 垂足为 M

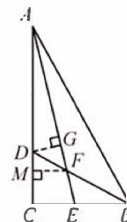
在 $Rt\triangle AMF$ 中

$$MF : AM : AF = 1 : 3 : \sqrt{10}$$

$$\therefore AM = \frac{6\sqrt{10}}{5}, MF = \frac{2\sqrt{10}}{5}$$

$$\because MF = CM$$

$$\therefore AC = AM + MF = \frac{8\sqrt{10}}{5}$$

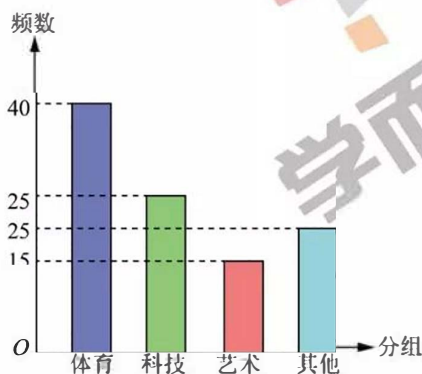


17. 解: 原式 $= 2 - 2 \times \frac{\sqrt{2}}{2} + \sqrt{2} + 1$
 $= 3$

18. 解: 原式 $= \frac{x-x+1}{x-1} \cdot \frac{(x+1)(x-1)}{(x+1)^2}$
 $= \frac{1}{x+1}$
 将 $x=2$ 代入, 得原式 $= \frac{1}{3}$

19. (1) 100, 0.25, 15

(2) 如图:



(3) $600 \times 0.15 = 90$ (人)

故全校喜欢艺术的学生大约 90 人.

20. (1) 证明: 由尺规作图可知.

AF 平分 $\angle BAC$, $AD = AE$, $DF = EF$

$\therefore \angle DAF = \angle EAF$

$\because DF \parallel AC$

$\therefore \angle DFA = \angle EAF$

$\therefore \angle DAF = \angle DFA$

$\therefore AD = DF$

$\therefore AD = DF = EF = AE$

\therefore 四边形 $ADFE$ 为菱形

$\because \angle BAC$ 与 $\angle DAE$ 重合, 点 F 在 BC 上

\therefore 菱形 $ADFE$ 为 $\triangle ABC$ 的亲密菱形.

(2) 设菱形的边长 a

即 $DF = AD = a$

则 $BD = 6 - a$

$\because DF \parallel AC$

$\therefore \triangle BDF \sim \triangle BAC$

$\therefore \frac{BD}{BA} = \frac{DF}{AC}$

即 $\frac{6-a}{6} = \frac{a}{12}$

$a = 4$

过 D 作 $DG \perp AC$, 垂足为 G

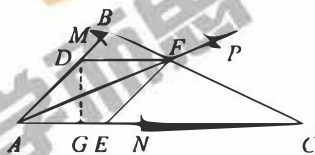
在 $\text{Rt}\triangle ADG$ 中

$\because \angle DAG = 45^\circ$

$\therefore DG = \frac{\sqrt{2}}{2} AD = 2\sqrt{2}$

$S_{\text{菱形}ADFE} = AE \cdot DG = 8\sqrt{2}$

\therefore 四边形 $ADFE$ 的面积为 $8\sqrt{2}$.



21. (1) 解: 设第一批饮料进价为 x 元

$$3 \times \frac{1600}{x} = \frac{6000}{x+2}$$

解, 得: $x = 8$

经检验, $x = 8$ 是原分式方程的解

所以第一批饮料的进价为 8 元.

(2) 解: 设销售单价为 y 元

第一批进货数量为 $1600 \div 8 = 200$ (件)

第二批进货数量为 $3 \times 200 = 600$ (件)

$200 \times (y - 8) + 600 \times (y - 8 - 2) \geq 1200$

解, 得: $y \geq 11$

所以销售单价至少是 11 元.

22.

【解析】解：(1) 过 A 作 $AF \perp BC$ ，垂足为 F ，交 $\odot O$ 于 G

$$\because AB = AC, AF \perp BC$$

$$\therefore BF = CF$$

Rt $\triangle ABF$ 中

$$AB = \frac{BF}{\cos B} = \sqrt{10}$$

(2) 连接 DG

$$\because AF \perp BC, BF = CF$$

$$\therefore AG \text{ 为 } \odot O \text{ 的直径}$$

$$\therefore \angle ADG = 90^\circ$$

$$\because \angle DAG = \angle EAF$$

$$\angle ADG = \angle AFE = 90^\circ$$

$$\therefore \triangle ADG \sim \triangle AFE$$

$$\frac{AD}{AF} = \frac{AG}{AE}$$

$$AD \cdot AE = AF \cdot AG$$

$$AD \cdot AE = AF \cdot AG$$

法一：连接 BG

$$\therefore \angle ABG = 90^\circ$$

$$\because BF \perp AG$$

$$\therefore BF^2 = AF \cdot FG$$

$$FG = \frac{1}{3}$$

$$AD \cdot AE = AF \cdot (AF + FG)$$

$$= 3 \times \frac{10}{3}$$

$$= 10$$

法二：连接 BO 设 $BO = AO = r$ ，则 $FO = 3 - r$ Rt $\triangle BOF$ 中

$$r^2 = 1^2 + (3 - r)^2$$

$$r = \frac{5}{3} \text{ 即 } AG = \frac{10}{3}$$

$$AD \cdot AE = AF \cdot AG = 10$$

法三：Rt $\triangle ABF$ 中

$$\sin \angle BAF = \cos \angle ABF = \frac{\sqrt{10}}{10}$$

$$\therefore \cos \angle BAF = \frac{3\sqrt{10}}{10}$$

连接 BG

$$\therefore \angle ABG = 90^\circ$$

$$AG = \frac{AB}{\cos \angle BAF} = \frac{10}{3}$$

$$AD \cdot AE = AF \cdot AG = 10$$

(3) 法一：在 BH 上截取 $MH = BH$



连接 AM 、 AD

$\because AH \perp BD$, $MH = DH$

$\therefore AM = AD$

$\because \angle ADM = \angle ACB$, $AB = AC$

$\therefore \triangle ABC \sim \triangle AMD$

$\therefore \angle BAC = \angle MAD$

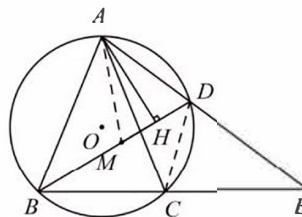
$\therefore \angle BAM = \angle CAD$

$\therefore \triangle ABM \cong \triangle ACD$ (SAS)

$\therefore BM = CD$

$\therefore BH = BM + MH$

$= DH + CD$



法二: 延长 BD 至点 N , 令 $DN = DC$

连接 AD 、 AN

$\because \angle ADB = \angle ACB = \angle ABC$

$\angle ADC + \angle ABC = 180^\circ$

$\angle ADN + \angle ADB = 180^\circ$

$\therefore \angle ADC = \angle ADN$

$\because AD = AD$, $CD = DN$

$\therefore \triangle ADC \cong \triangle ADN$

$\therefore AC = AN$

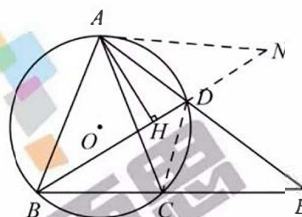
$\because AB = AC$

$\therefore AB = AN$

$\therefore AH \perp BN$

$\therefore BH = HN$

$= HD + CD$



法三: 过点 A 作 $AH' \perp CD$ 交 CD 的延长线于点 H' , 连接 AD ,

$\because \angle ADB = \angle ACB = \angle ABC$, $\angle ABC + \angle ADC = 180^\circ$,

$\angle ADC + \angle ADH' = 180^\circ$,

$\therefore \angle ADB = \angle ADH'$,

$\therefore \triangle ADH \cong \triangle ADH'$,

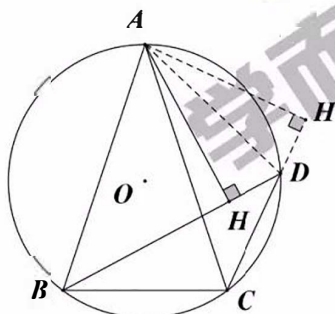
$\therefore AH = AH'$, $DH = DH'$,

又 $\because AB = AC$, $\angle ABD = \angle ACD$,

$\therefore \triangle ABH \cong \triangle ACH'$,

$\therefore BH = CH'$,

$\therefore BH = CH' = CD + DH' = CD + DH$, 得证.





23.

【解析】解：(1) 将 $B\left(-\frac{3}{2}, 2\right)$ 代入 $y=a\left(x-\frac{1}{2}\right)^2-2$ 得 $a=1$

\therefore 二次函数解析式为： $y=\left(x-\frac{1}{2}\right)^2-2$

$$\text{即 } y=x^2-x-\frac{7}{4}$$

(2) 法一：如图，易知： $A\left(\frac{1}{2}, -2\right), F\left(0, -\frac{7}{4}\right)$

$$\therefore k_{AF} = \frac{-\frac{7}{4}+2}{0-\frac{1}{2}} = -\frac{1}{2}$$

若 P 在 M 上方

$$\because \angle OP_1M = \angle MAF$$

$$\therefore OP_1 \parallel AF$$

$$\therefore k_{OP_1} = k_{AF} = -\frac{1}{2}$$

$$l_{OP_1}: y = -\frac{1}{2}x$$

$$\because A\left(\frac{1}{2}, -2\right) B\left(-\frac{3}{2}, 2\right)$$

$$\therefore l_{AB}: y = -2x - 1 \quad E(0, -1)$$

联立 AB, OP_1 得

$$-2x - 1 = -\frac{1}{2}x$$

$$x = -\frac{2}{3}$$

$$P_1\left(-\frac{2}{3}, \frac{1}{3}\right)$$

$$S_{\triangle OEP_1} = \frac{1}{2} \cdot OE \cdot |x_{P_1}| = \frac{1}{3}$$

若 P 在 M 下方

过 O 作 $OG \perp AB$ ，垂足为 G

$$\text{则 } l_{OG}: y = \frac{1}{2}x$$

联立 OG, AB 得

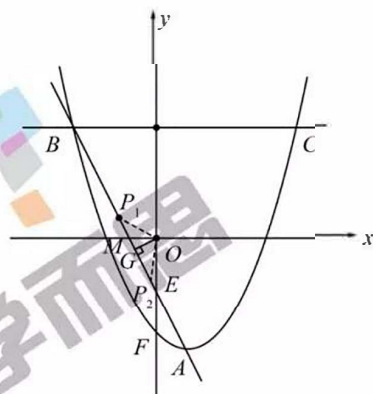
$$\frac{1}{2}x = -2x - 1, \quad x = -\frac{2}{5}$$

$$\therefore G\left(-\frac{2}{5}, -\frac{1}{5}\right)$$

$\because G$ 为 P_1, P_2 的中点

$$\therefore P_2\left(-\frac{2}{15}, -\frac{11}{15}\right)$$

$$S_{\triangle OEP_2} = \frac{1}{2} \cdot OE \cdot |x_{P_2}| = \frac{1}{15}$$



综上所述, $S_{\triangle OEP}$ 的面积为 $\frac{1}{3}$ 或 $\frac{1}{15}$.

法二: 易知 $A\left(\frac{1}{2}, -2\right)$ $B\left(0, -\frac{7}{4}\right)$

$$\therefore k_{AF} = -\frac{1}{2}$$

$$A\left(\frac{1}{2}, -2\right) \quad B\left(-\frac{3}{2}, 2\right)$$

$$\therefore k_{AB} = -2$$

$$\tan \angle MAF = \left| \frac{k_{AF} - k_{AB}}{1 + k_{AF} \cdot k_{AB}} \right| = \frac{3}{4}$$

$$\therefore \angle OPM = \angle MAF$$

$$\therefore \tan \angle OPM = \tan \angle MAF = \frac{3}{4}$$

$$\text{即 } \left| \frac{k_{AB} - k_{OP}}{1 + k_{AB} \cdot k_{OP}} \right| = \frac{3}{4}$$

$$\frac{-2 - k_{OP}}{1 - 2 \cdot k_{OP}} = -\frac{3}{4} \text{ 或 } \frac{-2 - k_{OP}}{1 - 2k_{OP}} = \frac{3}{4}$$

$$\text{解得 } k_{OP_1} = -\frac{1}{2} \quad k_{OP_2} = \frac{11}{2}$$

$$\therefore l_{OP_1}: y = -\frac{1}{2}x$$

$$l_{OP_2}: y = \frac{11}{2}x$$

l_{OP_1} 、 l_{OP_2} 分别与 l_{AB} 联立,

$$\text{得 } P_1\left(-\frac{2}{3}, \frac{1}{3}\right) \quad P_2\left(-\frac{2}{15}, -\frac{11}{15}\right)$$

$$\therefore S_{\triangle OEP_1} = \frac{1}{3} \quad S_{\triangle OEP_2} = \frac{1}{15}$$

综上所述, $\triangle OEP$ 的面积为 $\frac{1}{3}$ 或 $\frac{1}{15}$.

(3) 若 Q 在 AB 上运动, 如图:

$$\text{设 } Q(a, -2a-1)$$

$$\text{则 } QN = -2a \quad NE = -a$$

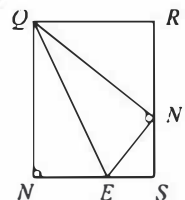
$$QN' = -2a$$

易知 $\triangle QRN' \sim \triangle N'SE$

$$\therefore \frac{QR}{N'S} = \frac{RN'}{ES} = \frac{QN'}{EN'} = 2$$

$$a = -\frac{5}{4}, \therefore Q\left(-\frac{5}{4}, \frac{3}{2}\right)$$

若 Q 在 BC 上运动, 且 Q 在 y 轴左侧, 如图:



设 $NE = a$, 则 $N'E = a$

易知, $RN' = 2$ $SN' = 1$

$QN' = QN = 3$

$\therefore QR = \sqrt{5}$ $SE = \sqrt{5} - a$

Rt $\triangle SEN'$ 中

$$(\sqrt{5} - a)^2 + 1^2 = a^2$$

$$a = \frac{3\sqrt{5}}{5}$$

$$\therefore Q\left(-\frac{3\sqrt{5}}{5}, 2\right)$$

若 Q 在 BC' 上运动, 且 Q 在 y 轴右侧, 如图:

设 $NE = a$, 则 $N'E = a$

易知, $RN' = 2$ $SN' = 1$

$QN' = QN = 3$

$\therefore QR = \sqrt{5}$ $SE = \sqrt{5} - a$

Rt $\triangle SEN'$ 中

$$(\sqrt{5} - a)^2 + 1^2 = a^2$$

$$a = \frac{3\sqrt{5}}{5}$$

$$\therefore Q\left(\frac{3\sqrt{5}}{5}, 2\right)$$

综上所述 $Q_1\left(-\frac{5}{4}, \frac{3}{2}\right)$, $Q_2\left(-\frac{3\sqrt{5}}{5}, 2\right)$, $Q_3\left(\frac{3\sqrt{5}}{5}, 2\right)$

