

2019秋·高一数学期中考试分析——八校

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第一部分

试卷参考答案

一、选择题

1	2	3	4	5	6
A	A	A	C	D	D
7	8	9	10	11	12
B	A	B	B	C	B

【12题解析】

12. 解：根据题意 $P = P_0 e^{-5k} = P_0 (1 - 20\%)$

$$\therefore e^{-5k} = 80\%$$

$$\therefore P_0 e^{-kt} \leq P_0 50\%$$

$$\therefore (e^{-5k})^{\frac{t}{5}} \leq \frac{1}{2}$$

$$\text{即 } (0.8)^{\frac{t}{5}} \leq \frac{1}{2}$$

$$\therefore t \geq 5 \log_{0.8} \frac{1}{2} = \frac{-5 \ln 2}{\ln \frac{4}{5}} = \frac{-5 \ln 2}{3 \ln 2 - \ln 10}$$

$\because \ln 2 = 0.69, \ln 10 = 2.30$, 代入解得 $t \geq 15$. 选 B.

二、填空题

13. $\frac{7}{2}$;

14. $(-\infty, 40]$;

15. 9;

16. $\left(-1, -\frac{3}{4}\right)$

【16题解析】

16. 解：化简得 $f(x) = x^2 + k$, 在 $(-\infty, 0)$ 单调递减;

$$\therefore \begin{cases} f(a) = b \\ f(b) = a \end{cases} \text{ 即 } \begin{cases} a^2 + k = b \\ b^2 + k = a \end{cases}$$

$$\therefore a^2 + k - (b^2 + k) = b - a, \text{ 整理得 } a + b = -1$$

$$\therefore a^2 + k = -1 - a, \text{ 即 } k = -a^2 - a - 1$$

$$\because a < b < 0, \therefore -1 < a < -\frac{1}{2}$$

$$\text{代入解得 } k \in \left(-1, -\frac{3}{4}\right).$$

三、解答题

17. 解:

$$(1) A \cup B = \{x \mid -2 < x < 3\}$$

(2) 由题意 $A \subseteq B$

$$\therefore \begin{cases} 2m \leq 1 \\ 1-m \geq 3 \end{cases}, \text{解得 } m \leq -2$$

$$\therefore m \in (-\infty, -2]$$

18. 解:

$$(1) f(x+1) = -(x+1)^2 + 4(x+1) - 3 = -x^2 + 2x, \quad x \in [-1, 2]$$

在 $(-1, 1)$ 单调增, $(1, 2)$ 单调递减,当 $x = -1$ 时有最小值 $f(x+1)_{\min} = -3$

$$(2) \text{令 } t = \log_3 x, t \in \left[-\frac{1}{2}, \frac{3}{2}\right]$$

则 $y = f(t) = -t^2 + 4t - 3$ 在 $\left[-\frac{1}{2}, \frac{3}{2}\right]$ 单调递增,当 $t = \frac{3}{2}$ 时有最大值, $y_{\max} = \frac{3}{4}$ 此时 $x = 3\sqrt{3}$.综上, 当 $x = 3\sqrt{3}$ 时, $y_{\max} = \frac{3}{4}$

19. 解:

$$(1) f(-1) = -f(1) = \frac{7}{3}$$

 $\because f(x)$ 为 \mathbf{R} 上的奇函数,

$$\therefore f(0) = 0$$

$$\text{当 } x < 0, -x > 0, \quad f(-x) = \frac{x}{3} - 2^{-x}$$

$$\therefore f(x) = -f(-x) = -\frac{x}{3} + 2^{-x}$$

$$\therefore f(x) = \begin{cases} -\frac{x}{3} - 2^x, & x > 0 \\ 0, & x = 0 \\ -\frac{x}{3} + 2^{-x}, & x < 0 \end{cases}$$

(2) ∵ $f(x)$ 为奇函数,

$$\therefore f(t^2 + 2t) + f(2t^2 - k) = f(t^2 + 2t) - f(k - 2t^2) < 0$$

即 $f(t^2 + 2t) < f(k - 2t^2)$ 对 $t \in \mathbb{R}$ 恒成立

∴ $f(x)$ 在 \mathbb{R} 上单调递减,

∴ $t^2 + 2t > k - 2t^2$ 恒成立

∴ $k < 3t^2 + 2t$ 恒成立

令 $g(t) = 3t^2 + 2t$, 当 $t = -\frac{1}{3}$ 时有最小值 $g(t)_{\min} = -\frac{1}{3}$

$$\therefore k \in \left(-\infty, -\frac{1}{3}\right)$$

20. 解:

$$(1) \text{令 } h(x) = f(x) - g(x) = \log_a(x+1) - \log_a(1-x)$$

∴ $x+1 > 0$ 且 $1-x > 0$

解得 $x \in (-1, 1)$

$$(2) \forall x \in (-1, 1), h(-x) = \log_a(-x+1) - \log_a(1+x) = -h(x)$$

∴ $h(x)$ 为奇函数.

(3) 当 $a=2$, $h(x) = \log_2(x+1) - \log_2(1-x) = \log_2\left(\frac{x+1}{1-x}\right)$ 为单调增函数.

证明: $\forall x_1, x_2 \in (-1, 1)$, $x_1 < x_2$

$$\therefore t_1 = \frac{x_1+1}{1-x_1} = -1 + \frac{2}{1-x_1}, t_2 = -1 + \frac{2}{1-x_2}$$

$$t_1 - t_2 = \frac{2}{1-x_1} - \frac{2}{1-x_2} = \frac{2(x_1 - x_2)}{(1-x_1)(1-x_2)} < 0$$

∴ $t_1 < t_2$, $\log_2 t_1 < \log_2 t_2$

$$\therefore h(x_1) - h(x_2) = \log_2 t_1 - \log_2 t_2 < 0$$

$$h(x_1) < h(x_2)$$

∴ $h(x)$ 在 $(-1, 1)$ 单调递增.

21.解：

$$(1) f(1)=f(-1)$$

$$2 + \frac{k}{2} = \frac{1}{2} + 2k, \text{ 解得 } k=1.$$

(2) 令 $t=2^x, t \in [m, m+2]$

$$f(x) \geq 4 \Rightarrow t + \frac{k}{t} \geq 4 \text{ 恒成立 } (t > 0)$$

$$\text{即 } k \geq (4t - t^2)_{\max}$$

$$\text{令 } g(t) = 4t - t^2, t \in [m, m+2]$$

$$\textcircled{1} 0 < m \leq 2, 2 < m+2 \leq 4$$

$$g(t) \text{ 在 } [m, m+2] \text{ 最大值为 } g(2)=4$$

$$\therefore k \geq 4$$

$$\textcircled{2} m > 2, g(t) \text{ 在 } [m, m+2] \text{ 单调减, 最大值为 } g(m)=4m-m^2$$

$$\therefore k \geq 4m-m^2$$

综上, 当 $0 < m \leq 2, k_{\min}=4$; 当 $m > 2, k_{\min}=4m-m^2$

22.解：

$$(1) f(x) = \sqrt{\frac{1-x^2}{1+x^2}} + \sqrt{\frac{1+x^2}{1-x^2}} = \frac{2}{\sqrt{1-x^4}}, x \in (-1, 1)$$

$$\therefore f(x)_{\min} = f(0) = 2$$

(2) $f(x)$ 为偶函数, 在 $(0, 1)$ 单调增, 在 $(-1, 0)$ 单调减;

证明:

$$\textcircled{1} \forall x_1, x_2 \in (-1, 0), x_1 < x_2$$

$$\begin{aligned} f(x_1) - f(x_2) &= \frac{2}{\sqrt{1-x_1^4}} - \frac{2}{\sqrt{1-x_2^4}} = \frac{2(\sqrt{1-x_2^4} - \sqrt{1-x_1^4})}{\sqrt{(1-x_1^4)(1-x_2^4)}} \\ &= \frac{2(x_1^4 - x_2^4)}{(\sqrt{1-x_2^4} + \sqrt{1-x_1^4})\sqrt{(1-x_1^4)(1-x_2^4)}} \end{aligned}$$

$$\because x_1^4 > x_2^4, \therefore f(x_1) - f(x_2) > 0$$

$$\therefore f(x_1) > f(x_2)$$

$f(x)$ 在 $(-1, 0)$ 单调减.

② $\forall x_1, x_2 \in (0,1)$, $x_1 < x_2$

$$f(x_1) - f(x_2) = \frac{2}{\sqrt{1-x_1^4}} - \frac{2}{\sqrt{1-x_2^4}} = \frac{2(\sqrt{1-x_2^4} - \sqrt{1-x_1^4})}{\sqrt{(1-x_1^4)(1-x_2^4)}}$$

$$= \frac{2(x_1^4 - x_2^4)}{(\sqrt{1-x_2^4} + \sqrt{1-x_1^4})\sqrt{(1-x_1^4)(1-x_2^4)}}$$

$$\because x_1^4 < x_2^4, \therefore f(x_1) - f(x_2) < 0$$

$$\therefore f(x_1) < f(x_2)$$

$f(x)$ 在 $(0,1)$ 单调增.

(3) 令 $t = \sqrt{\frac{1-x^2}{1+x^2}}$, 则 $t \in \left[\frac{1}{3}, 1\right]$

令 $y = t + \frac{a}{t}$, $t \in \left[\frac{1}{3}, 1\right]$, 由题意有: $2y_{\min} > y_{\max}$

$$\textcircled{1} 0 < a \leq \frac{1}{9},$$

$y = t + \frac{a}{t}$ 在 $\left[\frac{1}{3}, 1\right]$ 单调增,

$$y_{\min} = \frac{1}{3} + 3a, y_{\max} = 1 + a$$

$$\therefore \frac{2}{3} + 6a > 1 + a, \text{ 解得 } a > \frac{1}{15}$$

$$\therefore \frac{1}{15} < a \leq \frac{1}{9}$$

$$\textcircled{2} \frac{1}{9} < a \leq \frac{1}{3},$$

$y = t + \frac{a}{t}$ 在 $\left(\frac{1}{3}, \sqrt{a}\right)$ 单调减, 在 $(\sqrt{a}, 1)$ 单调增,

$$y_{\min} = 2\sqrt{a}, y_{\max} = \left\{1 + a, \frac{1}{3} + 3a\right\} = a + 1$$

$$\therefore 4\sqrt{a} > 1 + a, \text{ 解得 } 7 - 4\sqrt{3} < a < 7 + 4\sqrt{3}$$

$$\therefore \frac{1}{9} < a \leq \frac{1}{3}$$

$$\textcircled{3} \frac{1}{3} < a < 1$$

$y = t + \frac{a}{t}$ 在 $\left(\frac{1}{3}, \sqrt{a}\right)$ 单调减, 在 $(\sqrt{a}, 1)$ 单调增,

$$y_{\min} = 2\sqrt{a}, y_{\max} = \left\{1 + a, \frac{1}{3} + 3a\right\} = \frac{1}{3} + 3a$$

$$\therefore 4\sqrt{a} > \frac{1}{3} + 3a, \text{ 解得 } \frac{7 - 4\sqrt{3}}{9} < a < \frac{7 + 4\sqrt{3}}{9}$$

$$\therefore \frac{1}{3} < a < 1$$

④ $a \geq 1$,

$y = t + \frac{a}{t}$ 在 $\left[\frac{1}{3}, 1\right]$ 单调减,

$$y_{\min} = 1 + a, y_{\max} = \frac{1}{3} + 3a$$

$$\therefore 2 + 2a > \frac{1}{3} + 3a, \text{ 解得 } a < \frac{5}{3}$$

$$\therefore 1 < a < \frac{5}{3}$$

综上所述, $a \in \left(\frac{1}{15}, \frac{5}{3}\right)$