1~5：B C D C C　　6~10：D D C B A　　11~12：D C

12：$|\vec{a}+x\vec{b}|\geq|\vec{a}+\vec{b}|$

$\therefore \vec{a}^2+x^2\vec{b}^2+2x\vec{a}\cdot\vec{b}\geq\vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b}$

$\therefore x^2+2\sqrt{3}\cos\theta\cdot x-2\sqrt{3}\cos\theta-1\geq 0$

$\Delta=(2\sqrt{3}\cos\theta)^2+4(2\sqrt{3}\cos\theta+1)\leq 0$

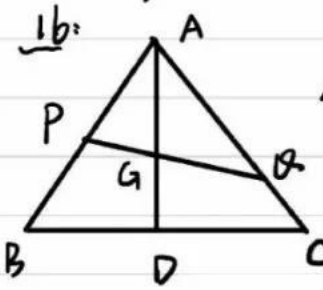
即 $12\cos^2\theta+8\sqrt{3}\cos\theta+4\leq 0$

$\therefore (\sqrt{3}\cos\theta+1)^2\leq 0$　　故 $\cos\theta=-\frac{\sqrt{3}}{3}$

又 $\theta\in[0,\pi]$，$\therefore \sin\theta=\frac{\sqrt{6}}{3}$，故 $\begin{cases}\sin 2\theta=2\sin\theta\cdot\cos\theta=-\frac{2\sqrt{2}}{3}\end{cases}$

13：$\frac{2\pi}{3}$　　14．$\frac{33}{13}$　　15．$5\sqrt{7}$　　16．2或$\frac{2}{5}$

16：



$\because \overrightarrow{AP}=\lambda\overrightarrow{AB}$，$\overrightarrow{AQ}=\mu\overrightarrow{AC}$

$\overrightarrow{AM}=\frac{2}{3}\overrightarrow{AD}=\frac{1}{3}(\overrightarrow{AB}+\overrightarrow{AC})=\frac{1}{3}\overrightarrow{AB}+\frac{1}{3}\overrightarrow{AC}=\frac{1}{3\lambda}\overrightarrow{AP}+\frac{1}{3\mu}\overrightarrow{AQ}$

由 P，G，Q 共线，$\therefore \frac{1}{3\lambda}+\frac{1}{3\mu}=1$，$\therefore \frac{1}{\lambda}+\frac{1}{\mu}=3$

$$\frac{S_{\triangle APQ}}{S_{\triangle ABC}}=\frac{\frac{1}{2}AP\cdot AQ\cdot\sin A}{\frac{1}{2}\cdot AB\cdot AC\cdot\sin A}=\frac{AP\cdot AQ}{AB\cdot AC}=\lambda\mu=\frac{4}{5}$$

即 $\begin{cases}\frac{1}{\lambda}+\frac{1}{\mu}=3\\ \lambda\mu=\frac{4}{5}\end{cases}$　$\therefore \begin{cases}\lambda=2\\ \mu=\frac{2}{5}\end{cases}$ 或 $\begin{cases}\lambda=\frac{2}{5}\\ \mu=2\end{cases}$

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17. 解：(1) 由 $\vec{a}\parallel\vec{b}$，∴ $\frac{3}{2}x=18$　∴ $x=12$，$\vec{b}=(9,12)$

由 $\vec{a}\perp\vec{c}$，∴ $\vec{a}\cdot\vec{c}=0$，即 $6+xy=0$，∴ $y=-3$，$\vec{c}=(4,-3)$

(2) $\vec{m}=4\vec{a}-\vec{b}=(-3,-4)$，$\vec{n}=2\vec{a}+\vec{c}=(7,1)$

∴ $\cos\langle\vec{m},\vec{n}\rangle=\frac{\vec{m}\cdot\vec{n}}{|\vec{m}|\cdot|\vec{n}|}=\frac{-25}{25\sqrt{2}}=-\frac{\sqrt{2}}{2}$

∴ $\langle\vec{m},\vec{n}\rangle=\frac{3\pi}{4}$

18. 解：(1) $2\cos^2\frac{A}{2}+\cos A=0$　∴ $1+\cos A+\cos A=0$

∴ $\cos A=-\frac{1}{2}$，由 $A=\frac{2\pi}{3}$

(2) $a^2=b^2+c^2-2bc\cdot\cos A=(b+c)^2-2bc(1+\cos A)$

∴ $17=25-bc$，$bc=8$

∴ $S_{\triangle ABC}=\frac{1}{2}bc\sin A=2\sqrt{3}$

19. 解：(1) ∵ $2ac\cos B=2c+b$　∴ $2\sin A\cos B=2\sin C+\sin B=2\sin(A+B)+\sin B$

∴ $2\sin A\cos B=2\sin A\cos B+2\cos A\sin B+\sin B$

∴ $(1+2\cos A)\cdot\sin B=0$

由 $\sin B>0$，得 $\cos A=-\frac{1}{2}$，∴ $A=\frac{2\pi}{3}$

∵ $c=4$，$b=2$，∴ $a^2=b^2+c^2-2bc\cdot\cos A=28$，得 $a=2\sqrt{7}$

(2) $|\vec{AD}|^2=(\frac{1}{3}\vec{AB}+\frac{2}{3}\vec{AC})^2=\frac{1}{9}\vec{AB}^2+\frac{4}{9}\vec{AC}^2+\frac{4}{9}\vec{AB}\cdot\vec{AC}$

$=\frac{16}{9}$

∴ $|\vec{AD}|=\frac{4}{3}$

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20. 解：(1) a, b, c 成等比数列，则 $b^2 = a \cdot c$

$\therefore \sin^2 B = \sin A \cdot \sin C = \frac{3}{4}$

$\therefore \sin B = \frac{\sqrt{3}}{2}$，又 B 为锐角，则 $B = \frac{\pi}{3}$

(2) $\vec{m} \cdot \vec{n} = \cos 2A - \frac{12}{5}\cos A = 2\cos^2 A + 1 - \frac{12}{5}\cos A$

$= 2\left(\cos A - \frac{3}{5}\right)^2 - \frac{43}{25}$

$\therefore$ 当 $\cos A = \frac{3}{5}$ 时，$\vec{m} \cdot \vec{n}$ 取得最小值

又 $\cos C = \cos\left(A + \frac{\pi}{3}\right) = \sin A \sin\frac{\pi}{3} - \cos A \cdot \cos\frac{\pi}{3} = \frac{4\sqrt{3}-3}{10} > 0$

故 $C \in \left(0, \frac{\pi}{2}\right)$，故此三角形为锐角三角形

21. 解：(1) 在 $Rt\triangle OBE$ 中，$OE = \frac{OB}{\cos\alpha} = \frac{25}{\cos\alpha}$

在 $Rt\triangle OAF$ 中，$\angle AFO = \alpha$，$OF = \frac{AO}{\sin\alpha} = \frac{25}{\sin\alpha}$

$\therefore EF = \sqrt{OE^2 + OF^2} = 25\sqrt{\frac{1}{\sin^2\alpha} + \frac{1}{\cos^2\alpha}} = \frac{25}{\sin\alpha \cdot \cos\alpha}$

$\therefore$ 周长 $l = OE + OF + EF = 25\left(\frac{1}{\cos\alpha} + \frac{1}{\sin\alpha} + \frac{1}{\sin\alpha \cdot \cos\alpha}\right)$

$= \frac{25(\sin\alpha + \cos\alpha + 1)}{2\sin\alpha\cos\alpha}$

当 F 与 D 重合时，α 最小，$\sin\alpha = \sin\angle ADO = \frac{AO}{OD} = \frac{1}{2}$，$\alpha = \frac{\pi}{6}$

当 E 与 C 重合时，α 最大，$\sin\alpha = \frac{BC}{OC} = \frac{\sqrt{3}}{2}$，$\alpha = \frac{\pi}{3}$

则 $\alpha \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

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(2) 费用最低时，即管道长度最短

$\because \sin\alpha+\cos\alpha=t$，则 $\sin\alpha\cdot\cos\alpha=\frac{t^2-1}{2}$

此时 $l=\frac{50}{t-1}$，$t=\sin\alpha+\cos\alpha=\sqrt{2}\sin(\alpha+\frac{\pi}{4})$

$\because \frac{\pi}{4}\le\alpha\le\frac{\pi}{2}$，

$\therefore \frac{3\pi}{4}\le\alpha+\frac{\pi}{4}\le\frac{3\pi}{4}$

则 $\frac{\sqrt{2}+1}{2}\le t\le\sqrt{2}$

$\therefore \frac{\sqrt{2}-1}{2}\le t-1\le\sqrt{2}-1$

$\therefore \sqrt{2}+1\le\frac{1}{t-1}\le\sqrt{2}+1$

故 $50(\sqrt{2}+1)\le l\le 50(\sqrt{2}+1)$

当 $\alpha+\frac{\pi}{4}=\frac{\pi}{2}$，即 $\alpha=\frac{\pi}{4}$，BE=25时

$l_{min}=50(\sqrt{2}+1)$

则此时最低费用为 $1500(\sqrt{2}+1)$ 元

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22. 解：(1) $g(x)=\cos x\cdot\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3}-(\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6})+\cos x$

$=\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x-\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x+\cos x$

$=-\sqrt{3}\sin x+\cos x$

$\therefore \overrightarrow{OM}=(-\sqrt{3},1)$

(2) $f(x)=\sqrt{3}\sin x+\cos x=2\sin(x+\frac{\pi}{6})$

由 $f(x)=\frac{6}{5}$，即 $\sin(x+\frac{\pi}{6})=\frac{3}{5}$

$\because -\frac{\pi}{6}<x<\frac{\pi}{3}$

$\therefore 0<x+\frac{\pi}{6}<\frac{\pi}{2}$

$\therefore \cos(x+\frac{\pi}{6})=\frac{4}{5}$

由 $\sin x=\sin(x+\frac{\pi}{6}-\frac{\pi}{6})$

$=\sin(x+\frac{\pi}{6})\cos\frac{\pi}{6}-\cos(x+\frac{\pi}{6})\cdot\sin\frac{\pi}{6}$

$=\frac{3}{5}\cdot\frac{\sqrt{3}}{2}-\frac{4}{5}\cdot\frac{1}{2}$

$=\frac{3\sqrt{3}-4}{10}$

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(3) $g(x)=\cos x-\sqrt{3}\sin x=2\cos(x+\frac{\pi}{3})$

$h(x)=2\cos[\frac{1}{2}(x-\frac{2\pi}{3})+\frac{\pi}{3}]=2\cos\frac{x}{2}$

假设存在 $P(x,2\cos\frac{x}{2})$ 使 $\vec{AP}\perp\vec{BP}$

即 $\vec{AP}\cdot\vec{BP}=0$

又 $\vec{AP}=(x+2,2\cos\frac{x}{2}-3)$，$\vec{BP}=(x-2,2\cos\frac{x}{2}-6)$

$\vec{AP}\cdot\vec{BP}=x^2-4+(2\cos\frac{x}{2}-3)(2\cos\frac{x}{2}-6)=0$

$\therefore x^2-4+4\cos^2\frac{x}{2}-18\cos\frac{x}{2}+18=0$

$\therefore 4(\cos^2\frac{x}{2}-\frac{9}{2}\cos\frac{x}{2})+x^2+14=0$

即 $4(\cos\frac{x}{2}-\frac{9}{4})^2+x^2=\frac{25}{4}$

$\because -1\le\cos\frac{x}{2}\le 1$

$\therefore -\frac{13}{4}\le\cos\frac{x}{2}-\frac{9}{4}\le-\frac{5}{4}$

$\therefore(\cos\frac{x}{2}-\frac{9}{4})^2\ge\frac{25}{16}$

故 $4(\cos\frac{x}{2}-\frac{9}{4})^2\ge\frac{25}{4}$

$\therefore x^2\le 0$，故 $x=0$

综上，存在 $P(0,2)$ 满足题意

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